Quantum Field Theory Course Program

Academic Year 2017-2018

1. 26/02/2018, 14.30 - 16.15.

   Aim and scope of the course. Description of the program and main references. Differences between Quantum Mechanics and Quantum Field Theories (QFT). Outlines on the perturbative formulation, operator formalism, path-integral formalism. Comments on ϕ^4 and Quantum Electrodynamics (QED). Triviality of ϕ^4 in $D = 4$ and its non-existence for $D > 4$. Non Borel summability of QED. Wigner's theorem and exact symmetries. Algebra of the observables and their *-automorphisms. The von-Neumann theorem and unitary equivalence of theories with finitely many degrees of freedom. Spontaneous symmetry breaking as a phenomenon in theories with infinitely many degrees of freedom. Formulation of QFT on the lattice. Axiomatic approach. Wightman's axioms and Wightman's reconstruction theorem. Euclidean formulation. Schwinger functions and Osterwalder-Schrader's reconstruction theorem.

   The excellent book by F. Strocchi, "Elements of QM of infinite systems", SISSA, Worlds Scientific, 1985, includes the analysis of some keys non-perturbative phenomena of QFT's. Spontaneous symmetry breaking, shortly illustrated during the lecture, is reported at pp. 115-120. Other useful references are `http://arxiv.org/pdf/1201.5459.pdf`, `http://arxiv.org/pdf/1502.06540.pdf` *and the two excellent books F. Strocchi, "An introduction to non-perturbative foundations of quantum field theory", Oxford, 2013, Haag, "Local quantum physics, fields, particles, algebras", Springer-Verlag, 1996. See also the notes: A brief introduction to different QFT approaches.*

2. 01/03/2018, 13.30 - 15.15.

   Review of the Dirac equation. Gamma's algebra and its representations. Covariance of the Dirac equation. Transformation of the spinors under the Poincaré group, $\psi'(x') = S(\Lambda)\psi(x)$. Representations of the Poincaré group. $S(\Lambda)$ in the case of parity transformations. Charge conjugation.

   Itzykson-Zuber, sect. 2-1-2, 2-1-3, 2-4-2. Further references are chapters 2 and 3 of Peskin-Schroeder, "Quantum Field Theory", and the chapters 11, 12 and 13 of Bjorken-Drell, "Relativistic Quantum Fields".

Notes: Unitary representation of the Poincaré Group - Wigner classification. Behaviour of local fields under the Poincaré group. Relativistic covariance. Finite-dimensional irreducible representations of the Lorentz group.

3. 08/03/2018, 13.30 - 15.15.

   Clifford Algebra and bilinear spinors. $S(\Lambda)$ in the case of charge conjugation. Discrete symmetries in the case of quantized fields. Parity operator.

   Itzykson-Zuber, sect. 3-4-1.

4. 12/03/2018, 14.30 - 16.15.

   Charge conjugation and time reversal. Transformation properties of the Dirac bilinears under P, C and T transformations. PCT theorem.

   Itzykson-Zuber, sect. 3-4-2, 3-4-3, 3-4-4. See also chapter 2 and 3 of Peskin-Schroeder, "Quantum Field Theory", and the chapters 11, 12 and 13 of Bjorken-Drell, "Relativistic Quantum Fields".

5. 15/03/2018, 13.30 - 15.15.

   Lehmann, Symanzik and Zimmerman reduction formula. Dirac formulation of the path integral.

   Sect. 5 of M. Srednicki, "Quantum Field Theory", Cambridge. See also the notes: Källen-Lehmann representation. N.B.: Srednicki uses the metric $g' = -g$, $diag(g') = (-1, 1, 1, 1)$. The scalar products defined by the two metrics have opposite sign. Ramond, sect. 2.1 e 2.2. Notes: On the Dirac paper where it has been first formulated the path integral. P. A. M. Dirac, "The lagrangian in quantum mechanics", Phys. Z. Sowjetunion **3** *(1933) 64. Reprinted in, Selected papers on quantum electrodynamics, J. Schwinger Ed., Dover, 1958. See also, `http://arxiv.org/pdf/quant-ph/0004090v1.pdf`. The standard text for the path integral is Feynman-Hibbs, "Quantum mechanics and path integrals", McGraw Hill, 1965, and the 2010 Dover edition commented by Styer.*

6. 19/03/2018, 14.30 - 16.15.

   Dirac paper on the formulation of the path integral. The need of the Hamiltonian formulation: the case $H = p^2 v(q)/2$.

   Ramond, sect. 2.2.

7. 22/03/2018, 13.30 - 15.15.

   Functional derivatives. Forced harmonic oscillator. Convergence methods: $i\epsilon$ prescription and euclidean formulation. Vacuum to vacuum amplitude in the presence of an external force.

   Ramond, sect. 2.3. Notes: Functional derivatives.

8. 26/03/2018, 14.30 - 16.15.

   Path integral for quadratic lagrangians. Bohm-Aharonov effect. Path integral in the case of scalar theories. Euclidean formulation.

   Felsager, "Geometry, Particles and Fields", Springer, 1998, pp. 51-56, pgg. 81-82. MacKenzie, sect. 4.1. Ramond, sect. 3.1, 3.2.

9. 29/03/2018, 13.30 - 15.15.

   Green's functions in the free case and their representation in momentum space. Feynman diagrams for the free theory. Feynman propagator. Legendre transform and effective action. Schwinger method to extract the potential from the path integral.

   Ramond, sect. 3.2 and 3.3. The rigorous proof that the path integral reproduces the N-point functions, topic not treated in the lectures, can be found in the notes. These follow Peskin-Schroeder, "An introduction to Quantum Field Theory", ABP 1995, pp. 282-284.

10. 05/04/2018, 13.30 - 15.15.

    Effective action in the general case. Schwinger-Dyson equations. Inverse of the Feynman propagator. Linked-cluster theorem and $Z[J]$ as generating functional of the connected Green's functions. Invariance of the N-point connected Green's functions under the shift $\phi(x) \to \phi(x) + f(x)$.

    Ramond sect. 3.3. Notes: $\phi_{\rm cl}(x)$ and the Schwinger-Dyson equation. $Z[J]$ as generator of the connected Green's functions (based in Kerson Huang, "Quantum Field Theory. From operators to path integrals", 2010. pp. 188-189. Notes: Comment on the connected Green functions.

11. 09/04/2018, 14.30 - 16.15.

    Saddle-point approximation. Determinants from gaussian integrals. Perturbative solution of the Klein-Gordon equation for ϕ^4 in the presence of an external current. Green functions at tree level. Topology of the Feynman diagrams.

    Ramond, sect. 3.4 and appendix A.

12. 12/04/2018, 13.30 - 15.15.

    Relation between the connected two-point function and $\Gamma^{(2)}$. Determinants and heat equation. Effective action at order $\hbar$. Dependence of the coupling constant on the mass scale (Coleman-Weinberg).

    Ramond, sect. 3.4 and 3.5. Notes: $\tilde{\Gamma}^{(2)}_E(p)\tilde{G}^{(2)}_{cE}(p) = 1$. *Comments on* $\Gamma[\varphi]$ *at order* $\hbar$.

13. 19/04/2018, 13.30 - 15.15.

    Breaking of dilatation symmetry by quantum effects. Perturbation theory and Feynman rules. Examples of Feynman diagrams: tadpole, setting sun, fish. Normal ordering singularities.

    Ramond, sect. 3.6 and 4.1.

14. 23/04/2018, 14.30 - 16.15.

    Loop expansion as power expansion in $\hbar$. Truncated Green functions and LSZ reduction formula. Superficial degree of divergence. Renormalizable, super-renormalizable and non-renormalizable theories. Weinberg theorem. Effective action as generating functional of proper vertices, Jona-Lasinio theorem.

    Ramond, sect. 4.2. Notes: Loop expansion as power expansion in $\hbar$*. Truncated green functions and LSZ reduction formula. Effective action as generating functional of proper vertices, Jona-Lasinio theorem. The discussion at pp. 111-112 of the Ramond book is also reported, with more care and clarity, in the Casalbuoni lectures: pp. 92-97 of* `http://theory.fi.infn.it/casalbuoni/dott1.pdf`*. See also pp. 139-142 of* `http://theory.fi.infn.it/casalbuoni/lezioni99.pdf` *and sect. 11.5 of Kleinert book, "Particles and Quantum Fields", World Scientific, 2016.*

15. 26/04/2018, 13.30 - 15.15.

    Jona-Lasinio theorem. Regularization methods. Dimensional regularization.

    Notes: Effective action as generating functional of proper vertices, Jona-Lasinio theorem. Ramond, sect. 4.3.

16. 03/05/2018, 13.30 - 15.15

    Dimensional regularization. Proof of the Feynamn parametrization formula. Euclidean action in 2ω dimension, dimensionless λ_{new} and 't Hooft mass parameter μ. Tadpole and fish diagrams.

    Ramond, sect. 4.3. The proof of the Feynman parametrization formula follows the one in `http://kodu.ut.ee/~kkannike/english/science/physics/notes/feynman_param.pdf`.

17. 07/05/2018, 14.30 - 16.15.

    Calculation of the fish and double scoop diagrams. Notes on the calculation of the setting sun diagram, analysis of the divergences, residue depending on the moment. Renormalization. On the Feynman rules. Mass term considered as a two-leg vertex, Feynman propagator with mass as diagrammatic series of the massless one with interaction given by the mass term. Counter-terms for $\tilde{\Gamma}^{(2)}(p)$ and $\tilde{\Gamma}^{(4)}(p)$. Recursive structure of the renormalization procedure.

    Ramond, sect. 4.4 and 4.5. Notes: On the Feynman rules. $\tilde{\Gamma}^{(2)}(p)$ *at one-loop with the counter-term contribution. A useful reference for further information is section 11 of the Kleinert text, "Particles and Quantum Fields", World Scientific, 2016.*

18. 10/05/2018, 13.30 - 15.15

    Renormalized lagrangian density. Relation between the bare and renormalized proper vertex functions. Renormalization group equation. Scale equation for $\tilde{\Gamma}^{(N)}_{\text{ren}}$. Bare parameters in terms of λ, m/μ ed ϵ. Renormalization prescriptions. 't Hooft and Weinberg prescriptions. The β function. Landau pole. Ultraviolet and infrared fixed points of β. Asymptotic freedom and confinement. Scaling of $\tilde{\Gamma}^{(N)}_{\text{ren}}$ and anomalous dimension.

    Ramond, sect. 4.5 and 4.6. Notes: Relation between the bare and renormalized proper vertex functions. Scaling of $\tilde{\Gamma}^{(N)}_{\text{ren}}$ *and anomalous dimension. The explicit steps concerning the equations 4.6.10 - 4.6.15 of the Ramond book are reported in the equations 31.11 - 31.23 of* `http://`

theory.fi.infn.it/casalbuoni/dott1.pdf. The english version is reported in Chapter 6 of http://theory.fi.infn.it/casalbuoni/lezioni99.pdf.

19. 14/05/2018, 14.30 - 16.15.

    Calculation of γ_m and γ_d. Vertex functions in the limit of large momenta in the case of a UV fixed point. Prescription dependence of the renormalization group coefficients. Prescription independence of the existence of a UV fixed point of the β function. Grassmann algebra. Derivation and integration for anticommuting variables.

    Ramond, sect. 4.6 and 4.7. The prescription dependence of the renormalization group coefficients is also discussed at pp. 131-132 of http://theory.fi.infn.it/casalbuoni/dott1.pdf. The english version is reported in Chapter 6 of http://theory.fi.infn.it/casalbuoni/lezioni99.pdf. The Grassmann algebra is reported in L.H. Ryder, "Quantum Field Theory", 2nd Edition. 1996, sect. 6.7.

20. 17/05/2018, 13.30 - 15.15 Review of the properties of the self-energy, Green's functions and the vertex proper functions. Fermionic path integral.

    L.H. Ryder, "Quantum Field Theory", 2nd Edition. 1996, sect. 6.7.

21. 21/05/2018, 14.30 - 16.15.

    Gauge theories. Path integral for gauge fields. Gauge fixing. Propagator of the gauge field. Feynman and Landau gauges. Non covariant gauge for QED. Faddeev e Popov method. Feynman rules in the covariant gauge. The case of QED. Ward-Takahashi identities in QED.

    L.H. Ryder, sect. 7.1, 7.2 and 7.4.

22. 24/05/2018, 13.30 - 15.15

    Ward-Takahashi identities in QED. BRS transformations. Furry theorem. γ_5 in dimensional regularization.

    L.H. Ryder, "Quantum Field Theory", sec. 7.4, 7.5. A reference for the Furry theorem is the solution of problem 58.2 of Srednicki's book reported his solution manual "Quantum Field Theory: Problem Solutions", available at https://drive.google.com/file/d/0B0xb4crOvCgTM2x6QkhKREg edit. The Dirac algebra in arbitrary dimension is discussed in the book: Collins, "Renormalization". A good paper on the subject is the one by S. Weinzierl, Equivariant dimensional regularization, hep-ph/9903380, available at https://arxiv.org/pdf/hep-ph/9903380.pdf.

23. 28/05/2018, 14.30 - 16.15.

    Feynman rules for QED. Superficial degree of divergence. Divergent Feynman diagrams at one loop. Electron and photon self-energies. Vertex function.

    L.H. Ryder, sect. 9.5.

24. 31/05/2018, 13.30 - 15.15

    Counter-terms of QED at one loop. Renormalization at one-loop. Lamb Shift. Anomalous magnetic moment.

    L.H. Ryder, "Quantum Field Theory", sect. 9.5 and 9.6. Itzykson-Zuber, sect. 2-2-3.