Advanced Quantum Field Theory Course Program

Academic Year 2019-2020. Last updated 26.12.2019

1. 30/09/2019. 14.30-16.30

   Topics: Presentation of the course. General aspects of quantum field theories.

   Aim and scope of the course. Description of the program and main references. Differences between Quantum Mechanics and Quantum Field Theories (QFT). Outlines on the perturbative formulation, operator formalism, path-integral formalism. Comments on ϕ_4^4 and QED. The triviality of ϕ_4^4. The non-existence of the interaction picture (Haag). Non-Borel summability of QED. The von-Neumann theorem and unitary equivalence of theories with finitely many degrees of freedom. Spontaneous symmetry breaking as a phenomenon in theories with infinitely many degrees of freedom. Formulation of QFT on the lattice. Axiomatic approach. Non-renormalizability of Quantum Gravity.

   References: the main points are also discussed in the QFT Lecture Notes `https://www2.pd.infn.it/~matone/QFTCourseNotes.pdf`. Excellent references for the above topics are the books by F. Strocchi, "Elements of QM of infinite systems", SISSA, Worlds Scientific, 1985, the recent "Symmetry Breaking in the Standard Model: A Non-Perturbative Outlook", SNS, Spinger, 2019 and "Symmetry Breaking", whose third edition will appear soon. Other useful references are `http://arxiv.org/pdf/1201.5459.pdf`, `http://arxiv.org/pdf/1502.06540.pdf` and the two excellent books F. Strocchi, "An introduction to non-perturbative foundations of quantum field theory", Oxford, 2013, Haag, "Local quantum physics, fields, particles, algebras", Springer-Verlag, 1996.

2. 01/10/2019. 14.30-16.30

   Topics: Coleman-Mandula theorem. Bogoliubov transformations.

   Witten proof of the Coleman-Mandula theorem. Bogoliubov transformations and their singularities in the infinite volume limit.

   Reference: The Witten proof of the Coleman-Mandula is reported, for example, in M. Shifmann, "Advanced Topics in Quantum Field Theory", pp. 413-415.

3. 07/10/2019. 14.30-16.30

   Topics: Free electron gas. Wigner and von Neumann theorems and spontaneous symmetry breaking.

   The Baker-Campbell-Hausdorff (BCH) formula in QM and QFT. Closed forms of the BCH formula. Free electron gas and Valatin-Bogoliubov transformations (F. Strocchi, "Elements of QM of infinite systems", pp. 69-73). Non-unitarily equivalent representations. Wigner theorem and exact symmetries. von Neumann theorem and non-existence of spontaneous symmetry breaking in quantum theories with finitely many degrees of freedom (F. Strocchi, "Elements of QM of infinite systems", pp. 115-118).

4. 08/10/2019. 14.30-16.30

   Topics: Spontaneous symmetry breaking. Representations of the Lorentz Group.

   Spontaneous symmetry breaking and the Standard Model. Representations of the Lorentz group.

   References: SSB in the SM: Cheng and Li, "Gauge Theory Of Elementary Particle Physics". Representation of the Lorentz group: `https://www2.pd.infn.it/~matone/QFTCourseNotes.pdf`.

5. 14/10/2019. 14.30-16.30

   Topics: Representations of the Lorentz group. Spinorial representations. Dotted and undotted spinors.

   $SL(2, C)$ transformations uniquely expressed as the product of a Hermitian and a unitary matrix. The metric and Ricci tensors as the only invariants under Lorentz transformations. Tensors expressed as a sum of irreducible representations. Dotted and undotted spinors.

   References: all the topics are covered in the QFT Lecture Notes `https://www2.pd.infn.it/~matone/QFTCourseNotes.pdf`. The presentation of the dotted and undotted notation for spinors has been extended by following the beautiful notes by N.Cabibbo, "Relatività - Teoria di Dirac", `http://chimera.roma1.infn.it/NICOLA/poincare.pdf`. Other references include A. Bilal, "Introduction to Supersymmetry", hep-th/0101055, `https://arxiv.org/pdf/hep-th/0101055.pdf` M. Bertolini, "Lectures on Supersymmetry", `https://people.sissa.it/~bertmat/susycourse.pdf` and D. Cassani, "Introduction to Supersymmetry", `https://userswww.pd.infn.it/~cassani/susy_advanced_topics.pdf`.

6. 15/10/2019. 14.30-16.30

   Topics: ϕ^3_6.

   Superficial degree of divergence. The dimension of the coupling constants and renormalizability. Lehmann-Symanzik-Zimmermann reduction formula and free parameters in the Lagrangian density. General properties of ϕ^3_6. Renormalized Lagrangian density. Feynman diagrams contributions to the generating functional $Z[J]$. Calculation of the combinatorics of the Feynman diagrams and symmetry factors.

   References: the lectures on ϕ^3_6 is the only part of the course that does not follow standard available literature. The presentation followed private notes by K. Lechner, mainly based on the book by J.C. Collins, "Renormalization". However, the notes of all the course have been taken by G.B. Carollo and colleagues and, although still very preliminary, such notes can be used as a guideline.

7. 21/10/2019. 14.30-16.30

   Topics: 1-loop corrections for ϕ^3_6.

   1-loop contributions to the propagator and vertex function to ϕ^3_6. Counterterms.

8. 22/10/2019. 14.30-16.30

   Topics: Renormalization group of ϕ^3_6.

   Beta function of ϕ^3_6. Running coupling constant. Calculation of γ_m and γ_d up to order g^2. The renormalization group equation. The anomalous dimension.

9. 28/10/2019. 14.30-16.30

   Topics: Beta function and higher order renormalization of ϕ^3_6.

   Coupling constant as function of the scaling parameter s. The Λ scale in ϕ^3_6 and QCD. The case of QED and ϕ^4_4. Check of the scaling relation for the anomalous dimension. Polynomial dependence on the external momenta of the divergent parts of the 1PI. Renormalization of ϕ^3_6 at higher order.

10. 04/11/2019. 14.30-16.30

    Topics: Renormalizability of ϕ_6^3. QED.

    Renormalizability of ϕ_6^3. Calculation of divergent diagrams. Overlapping and nested divergences. Reducing the ultraviolet singularities by taking the derivatives with respect to the external momenta. QED. Ward Identities.

11. 05/11/2019. 14.30-16.30

    Topics: Renormalizability of QED.

    Renormalizability of QED. Skeleton graphs. Reducible and irreducible graphs. Equivalence of renormalization by subtraction and multiplicative renormalization.

    Reference: L.H. Ryder, "Quantum Field Theory".

12. 11/11/2019. 14.30-16.30

    Topics: Yang-Mills theories. BRS transformations and Slavnov-Taylor identities.

    Short review of Yang-Mills theories, Faddeev-Popov ghosts, BRS transformations and Slavnov-Taylor identities.

    Reference: L.H. Ryder, "Quantum Field Theory". Warning: in the book there is a mistake concerning the Legendre transformation, see course notes taken by Carollo.

13. 12/11/2019. 14.30-16.30

    Topics: Beta function for Yang-Mills theories.

    One loop contributions of the beta function in Yang-Mills theories. Running coupling constant. Asymptotic freedom.

    Reference: L.H. Ryder, "Quantum Field Theory".

14. 18/11/2019. 14.30-16.30

    Topics: Topological structures in Field Theory.

    The Sine-Gordon model. Kink solutions. Conserved charge of topological origin. Finite energy solutions. Comments on solitons and the Korteweg-de Vries equation. Higher dimensional generalizations: looking for vortex lines. Divergence of the energy of topological solutions and the need of adding a gauge field.

    Reference: L.H. Ryder, "Quantum Field Theory".

15. 19/11/2019. 14.30-16.30

    Topics: Kibble-Higgs Lagrangian, SSB and topological phenomena.

    The Kibble-Higgs Lagrangian density. Problems with the semiclassical approximation formulation of Spontanous Symmetry Breaking. Elitzur theorem. The stable vortex solutions by Nielsen and Olsen. Topological structures.

    References: The proof of Elitzur theorem follows the one in F. Strocchi, "Elements of QM of infinite systems", pp. 162-167. The reference for the Nielsen and Olsen solution and the topological structures is L.H. Ryder, "Quantum Field Theory".

16. 02/12/2019. 14.30-16.30

    Topics: Dirac Monopole.

    Dirac monopole and Dirac string. Charge quantization. Fiber bundles and absence of singularity. Fiber bundles and holomorphic sections on the Riemann sphere. The Wu-Yang solution.

    Reference: L.H. Ryder, "Quantum Field Theory".

17. 03/12/2019. 14.30-16.30

    Topics: 't Hooft-Polyakov Monopoles.

    Homotopy groups. Monopoles and the 't Hooft-Polyakov models. The case of the $O(3)$ gauge group. Brouwer degree of the mapping from S^2 to S^2. Absence of monopoles in the Weinbegr-Salam theory.

    Reference: L.H. Ryder, "Quantum Field Theory".

18. 09/12/2019. 14.30-16.30

    Topics: Instantons in $SU(2)$ gauge theories.

    The $SU(2)$ gauge connection in Euclidean space. $\pi_3(S^3) = \mathbb{Z}$. The dual tensor and $\mathrm{Tr}(F\tilde{F})$ as four divergence. The field equations.External derivatives and Stokes theorem. The Pontryagin index. Euclidean action for bound states in Quantum Mechanics and WKB approximation. Brouwer degree of the $S^3 \to S^3$ map.

    Reference: L.H. Ryder, "Quantum Field Theory".

19. 10/12/2019. 14.30-16.30

    Topics: Instantons and tunneling. Supersymmetry.

    Tunneling and instantons in quantum mechanics. Theta vacua. General properties of supersymmetric theories. Super-Poincaré group.

    References: L.H. Ryder, "Quantum Field Theory". The standard reference for supersymmetryc theories is J. Wess and J. Bagger, "Supersymmetry and Supergravity". The lectures follow the reviews by Bilal, Bertolini and Cassani.

20. 16/12/2019. 14.30-16.30

    Topics: Representations of the supersymmetric algebra.

    Representations of the supersymmetric algebra. P^2 as unique Casimir operator. Supermultiplets. Massless supermultiplets. Clifford vacuum.

    References: the reviews by Bilal, Bertolini and Cassani.

21. 17/12/2019. 14.30-16.30

    Topics: Supermultiplets. Superspace and superfields.

    Massless and massive supermultiplets for $N = 1, 2, 4$ and 8. Superspace and superfields. Basic identities in Grassmaniann calculus. Supercharges as generators of translations in the Grassmann parameters.

    References: the reviews by Bilal, Bertolini and Cassani.

22. 07/01/2020. 14.30-16.30

    Topics: Chiral superfields. Wess-Zumino action. $N = 2$ SYM theory and Seiberg-Witten theory.

    Covariant derivatives and chiral superfields. Construction of supersymmetric invariant actions. Wess-Zumino action. Gauge transformations. θ-angle. $N = 2$ SYM theory. Holomorphicity of the prepotential. The case of gauge group SU(2). SSB and Wilsonian action.

23. 13/01/2020. 14.30-16.30

    Topics: Seiberg-Witten theory.

    Dual formulation of $N = 2$ SYM theory. Asymptotic freedom and the singularities in the moduli space of quantum vacua $\mathcal{M}$ for large u. The monodromy matrix at infinity. Singularity in $\mathcal{M}$ where the monopole becomes massless. T^2 invariance and dual formulation. The $\Gamma(2)$ symmetry group from the dual formulation.

24. 13/01/2020. 14.30-16.30

    Topics: Solution of the Seiberg-Witten theory.

    The three singularities in the moduli space of quantum vacua. Short introduction to uniformization theory: the uniformizing map and uniformizing equation. Effective coupling constant as the inverse of the uniformizing map of the thrice punctured Riemann sphere Σ_3. The Poincaré metric on the upper half-plane and on Σ_3. The Liouville equation and the Liouville stress tensor. Isomorphism between the fundamental group of Σ_3 and the uniformizing group $\Gamma(2)$. Hilbert-Einstein action and Euler characteristic. Accessory parameters. Non-perturbative relation between the u-modulus and the prepotential. Instanton recursion relations.

Below is reported some literature on Seiberg-Witten theory. The lectures on this topic followed Refs. 2 and 3. The part concerning the uniformization and Liouville theories used in Ref. 3 can be found in Ref. 4, from Section 2.2 up to Section 2.6.

1. Seiberg-Witten paper [here]

2. Bilal review [here]

3. Non-perturbative relation between $u = \langle \mathrm{Tr}\,\phi^2 \rangle$ and $\mathcal{F}$. Instanton recursion relations [here]

4. Uniformization and Liouville theories [here]

5. Proof of the u-$\mathcal{F}$ relation from instanton calculus [here]

6. Proof of the Seiberg-Witten conjecture [here]

7. Exact β-function [here]

8. Analysis of the Wilsonian action [here]

Excellent references for instanton calculus in Seiberg-Witten theory are

1. Review by Khoze, Mattis and Slater [here]

2. Review by Dorey, Hollowood, Khoze and Mattis [here]

3. Travaglini PhD thesis [here]

4. Slater PhD thesis [here]