An Effective Description of the Landscape

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Quantum Field Theories with simple vacuum structure and limited number of fields: integrate out heavy fields and obtain an effective quantum action for the light modes

Practically impossible to do that in theories with a very large (or infinite) degrees of freedom and a tremendously complicated vacuum structure

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Common procedure: ignore most of the heavy fields

We will also ignore most heavy states.
We will assume that string massive states, Kaluza-Klein and winding excitations are all negligible, so that the “microscopic” theory is four-dimensional.

More precisely, we will consider 4D SUGRA theories with N=1 SUSY

Despite above huge simplification, SUGRA theories contains typically **hundreds** of fields (moduli associated to the underlying internal geometry) and hence concrete studies still very hard

Aim

Study under what conditions massive chiral multiplets can be neglected (or “frozen”) and yet one can trust the resulting simple effective theory

Plan

- Warm-up: definition of heavy fields and non-SUSY case
- Pure F-term analysis and no charged fields
- Charged fields and gauge dynamics
- Summary of results

Non-linear, non-SUSY sigma model

We will not consider absolutely general theories, but only a subset where one can hope to freeze some fields

This implies that the potential is not totally arbitrary

$$\mathcal{L} = \frac{1}{2} g_{MN}(\phi^M) \partial\phi^M \partial\phi^N - V(\phi^M)$$

$$V(\phi^M) = V_0(H^i) + V_1(H^i, L^\alpha)$$

$M = (i, \alpha)$, $\phi^M = (H^i, L^\alpha)$. We study the theory in the (H, L) configuration space region where

$$|V_1| \ll |V_0|$$

The splitting between the fields H^i and L^α is dictated by V_0.

$V_1 \to \epsilon V_1$, with $\epsilon \ll 1$.

$$\epsilon \simeq \frac{m_L}{m_H}$$

Assume at $\langle\phi^M\rangle$ metric g_{MN} is non-singular and $\partial_i\partial_j V_0$ positive definite with all eigenvalues parametrically larger than ϵ. We show that

$$\mathcal{L}_{sim} = \frac{1}{2} g_{\alpha\beta}(L^\alpha, H_0^i)\partial L^\alpha \partial L^\beta - \Big[V_0(H_0^i) + \epsilon V_1(H_0^i, L^\alpha)\Big]$$

provides correct effective Lagrangian for arbitrary kinetic mixing terms at leading order in an expansion in ϵ.

$$H_0^i : \partial_i V_0(H_0) + \mathcal{O}(\partial^2\phi) = 0$$

Starting point: study the vacua in the original non-canonical field basis

$$O(\epsilon^0) : \partial_i V_0(H_0^j) = 0,\ L^\alpha \text{ undetermined}$$

$$\begin{aligned} O(\epsilon) : \quad & \partial_\alpha V_1(H_0^i) = 0 \text{ gives } L_0^\alpha \\ & \partial_i\partial_j V_0(H_0^k) H_1^j + \partial_i V_1(H_0^k, L_0^\alpha) = 0 \end{aligned}$$

gives the small displacement of H: $H = H_0 + \epsilon H_1$

Once vacuum (H_0, L_0) is found, look for canonical field fluctuations.

Interestingly, one can always go to a base where H are canonical heavy fields

Write $g_0 = (T^{-1})^t T^{-1}$ (Cholesky decomposition)

$$T = \begin{pmatrix} (T_H)^i_j & 0 \\ (T_{HL})^\alpha_i & (T_L)^\alpha_\beta \end{pmatrix} \qquad \Rightarrow \hat{\phi} = T\hat{\phi}_c, \quad \hat{\phi} = \phi - \langle\phi\rangle$$

In new basis, Lagrangian reads

$$\mathcal{L} = \left[\frac{1}{2} + O(\epsilon)\right]\left[(\partial\hat{H}_c)^2 + (\partial\hat{L}_c)^2\right] + \ldots + V_0(H_0 + \epsilon H_1 + T_H\hat{H}_c)$$

$$+\epsilon V_1(H_0 + \epsilon H_1 + T_H\hat{H}_c, L_0 + T\hat{\phi}_c) + O(\epsilon^2)$$

Due to triangular form of T, $\hat{H}^i_c$ are linear combinations of $\hat{H}^i$ only

$\Longrightarrow$

affect $\mathcal{L}$ at $O(\epsilon^2)$ only. Setting them to zero is OK.

Going back to non-canonically normalized fields just gives $g_{\alpha\beta}(L^\alpha, H_0^i)$

$$\mathcal{L}_{full} = \left[\frac{1}{2} g_{\alpha\beta}(L, H_0) + O(\epsilon)\right] \partial L^\alpha \partial L^\beta - \left[V_0(H_0 + \epsilon H_1) + \epsilon V_1(H_0, L) + O(\epsilon^2)\right]$$

At leading order

$$\mathcal{L}_{sim} = \mathcal{L}_{full}$$

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Final lesson: do not worry for kinetic mixing

and work with non-canonically normalized fields

Flat SUSY (no matter)

$$W(H,L) = W_0(H) + \epsilon W_1(H,L)$$

Kahler potential arbitrary, provided metric eigenvalues parametrically larger than ϵ

We now show that effective theory with frozen fields, defined by

$$W_{sim} = W(H_0, L), \qquad K_{sim} = K(H_0, L)$$

is reliable at $\mathcal{O}(\epsilon)$ in K and $\mathcal{O}(\epsilon^2)$ in W

Proper effective theory is obtained by integrating out chiral fields H

Use manifestly SUSY approach in super-fields

The super-field equations of motion are

$$\partial_H W + \mathcal{O}(\bar{D}^2 \partial_H K) = 0$$

Covariant derivatives terms will give rise to $D^n K$ terms in K_{eff}

$\Rightarrow F_L^n$ terms with $n > 2$

but $F_L = \partial_L W = \mathcal{O}(\epsilon) \Rightarrow$ all $D^n, \bar{D}^n$ terms can be neglected

Thus we have to solve $\quad \partial_H W = 0$

$$H^i = H_0^i + \epsilon H_1^i(L) + \mathcal{O}(\epsilon^2)$$

where

$$\partial_H W_0 = 0$$

$$
\begin{aligned}
W_{full} &= W_{sim} + \mathcal{O}(\epsilon^2) \\
K_{full} &= K_{sim} + \mathcal{O}(\epsilon)
\end{aligned}
$$

In particular $$V_{full} = V_{sim} + \mathcal{O}(\epsilon^3)$$

Notice: freezing of H has to be performed at the level of W and K and not of the component Lagrangian.

In other words, heavy field integration is not trivial, but leading terms automatically kept when freezing H in W and K

Backreaction of SUSY breaking induced by light fields on heavy ones is negligible:

$$F^L \sim O(\epsilon), \qquad F^H \sim O(\epsilon^2)$$

At leading order $$\mathcal{L}_{sim} = \mathcal{L}_{full}$$

Supergravity (no matter)

Requiring effective theory to be SUSY gives ($M_p = 1$)

$$F_{i,0} = \partial_i W_0 + (\partial_i K) W_0 = 0$$

Obstruction: for generic K, H_0 depends on $L \Longrightarrow$ freezing not well-defined

Useful way to estimate effect of gravitational corrections is obtained in a super-conformal approach. Requiring effective theory to be quadratic in the F implies that [Brizi, Gomez-Reino, Scrucca]

$$F_\Phi = e^{K/2}\left(\overline{W} + \frac{1}{3} K_M F^M\right) \ll m_H^2$$

In our case we have to require that $F_\Phi \simeq \mathcal{O}(\epsilon)$, $F_L \simeq \mathcal{O}(\epsilon)$

$$W_0 = \mathcal{O}(\epsilon)$$

Remarkably, all flat space analysis still applies, provided this condition holds

Yukawa couplings

Generalize form of superpotential

$$W = W_0(H) + \eta \widetilde{W}_0(H, M, Z, C) + \epsilon W_1(H, M, Z, C)$$

$$\begin{aligned} \widetilde{W}_0 &= Y(H, M, Z)C^3 + \mathcal{O}(C^4) \\ W_1 &= \widetilde{W}_1(H, M, Z) + \mu(H, M, Z)C^2 + \mathcal{O}(C^3) \end{aligned}$$

$$K = K_0 + K_1|C|^2 + (K_2 C^2 + c.c) + \mathcal{O}(C^3)$$

M: light moduli
Z: charged fields with $\mathcal{O}(1)$ VEV's
C: charged fields with $\mathcal{O}(\epsilon)$ VEV's.

Integration of heavy fields no longer negligible.
Find out up to what powers in the C's W_{sim} and K_{sim} are reliable

$$\partial_H W = 0$$

New induced superpotential couplings are of the form

$$O(\epsilon^0) : \eta^2 C^{N_i+N_j}$$

$$O(\epsilon) : \eta C^{N_i+2}$$

New induced Kähler couplings are of the form

$$O(\epsilon^0) : \eta(C^{N_i} + c.c.) + \eta^2 |C|^{N_i+N_j}$$

$$V(C)_{full} = V(C)_{sim} + \mathcal{O}(\epsilon^5)$$

$$|C| \lesssim \mathcal{O}(\epsilon)$$

Switch on gauge couplings

Assume $G \to H$ at a scale parametrically larger than ϵ

$$\delta\phi^M = \lambda^A X_A^M ,\, \delta\bar{\phi}^{\bar{M}} = \lambda^A \overline{X}_A^{\bar{M}}$$

$$D_A = i X_A^M K_M = -i \overline{X}_A^{\bar{M}} K_{\bar{M}}$$

$\partial_i W_0 = 0$ does *not* fix all H since gauge invariance requires

$$X_A^i \partial_i W_0 = 0$$

No well-defined notion of freezing apply to non-neutral heavy fields

$$\Longrightarrow \quad X_A^i = 0$$

Related mistakes performed by "freezing" non-neutral fields:

1) no gauge-invariant meaning
2) Not manifestly heavy: one or more linear combinations cannot admit mass terms
3) Even if somehow induced operators are negligible, related F-terms not suppressed

For non-linear realization where $\delta H = const.$, freezing of H can be interpreted as a possible gauge-fixing and eaten by heavy gauge field but its effect will reappear in the low-energy dynamics when vector field is integrated out [Arkani-Hamed, Dine, Martin; Binetruy et al.]

Important consequence: constant FI terms do not arise in string theory
$\Rightarrow$ they should be seen as field-dependent ones
$\Rightarrow$ dynamics of the associated field with $X \neq 0$ must always be studied.

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Important consequence: constant FI terms do not arise in string theory
$\Rightarrow$ they should be seen as field-dependent ones
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This is independent of gravity and is purely dictated by gauge invariance

Vector multiplets do not appear in W
$\Rightarrow$ SUSY integration of heavy chirals not affected at leading order

Holomorphic gauge kinetic functions reliable up to terms of $\mathcal{O}(C^N)$

When $G \to H$ heavy vector super field have to be integrated out

$$\partial_V K = 0$$

Fix a gauge (such as gauging away one chiral multiplet for each vector)

$$K' = K(V_0) \qquad W' = W(V_0)$$

$$K'_{sim} = K'(H_0) \qquad W'_{sim} = W'(H_0)$$

Notice: for realistic K's, $\partial_V K = 0$ *does not* admit approximately constant solutions $V = V_0$ $\Longrightarrow$ V cannot be frozen

Summary

$$W(H,M,Z,C) = W_0(H) + Y(H,M,Z)C^3 + \epsilon W_1(H,M,Z,C)$$

Look for H_0: $\partial_H W_0 = 0$. If

1. $m_H \gg \epsilon \; \forall H$
2. $\exp(K/2)W = \mathcal{O}(\epsilon)$, $W = \mathcal{O}(\epsilon)$
3. $|C| \sim \mathcal{O}(\epsilon)$

Heavy fields can be frozen.

Vector fields can then be integrated out in simple theory by taking

$$\partial_V K = 0$$

and gauge choice with $Z = Z_0$ and/or $M = M_0$ for some M and Z.

Gravity does not introduce any additional difficulty.

Application: IIB flux compactifications

Some debate on whether it is justified to first stabilize complex structure + dilaton and then Kahler moduli (KKLT procedure)

$$W_{KKLT} = W_0(S, Z) + \epsilon W_1(S, Z, T)$$

Definite answer: provided $W_0 \sim \epsilon$, no problem occurs, even for arbitrary Kähler mixing between all fields

Of course, W_0 has to provide a large positive SUSY mass term for all S, Z

Example: Non-trivial KKLT like SUGRA toy model

$$
\begin{aligned}
K &= -\log\left[(T+\bar{T}-\delta V_X)^{3/2}+\xi(S+\bar{S})^{3/2}\right]^2(Z+\bar{Z})(S+\bar{S}) \\
&\quad +\frac{\bar{\phi}\,e^{-2V_X}\,\phi}{(Z+\bar{Z})^{n_\phi}}+\frac{\bar{\chi}\,e^{2V_X}\,\chi}{(Z+\bar{Z})^{n_\chi}}\,, \\
W &= aZ^2+bZ+S(cZ^2+dZ+e)+mZ\phi\chi+\beta Z^2\phi^{\alpha\delta/2}e^{-\alpha T}
\end{aligned}
$$

Heavy fields: S (dilaton), Z (complex structure modulus)

T: universal Kähler modulus
ϕ, χ: two charged fields with opposite $U_X(1)$ charge

$U(1)_X$ holomorphic gauge kinetic function: $f_X = T$.
$\delta T = i\delta/2\Lambda$, $\delta\phi = i\Lambda\phi$, $\delta\chi = -i\Lambda\chi$.

$$
D_X = \frac{|\chi|^2}{(2Z_r)^{n_\chi}} - \frac{|\phi|^2}{(2Z_r)^{n_\phi}} + \frac{3\delta T_r^{1/2}}{4(T_r^{3/2}+\xi S_r^{3/2})}
$$

$$
\begin{aligned}
a &= -2.55 - 10^{-13}\,, \quad b = 25.5 + 2\cdot 10^{-12}\,, \quad c = 0.25\,, \quad d = -2.45\,, \quad e = -0.5 \\
\alpha &= 1\,, \;\; \beta = -0.5\,, \;\; \delta = 1\,, \quad m = 2.44\times 10^{-12}\,, \;\; n_\chi = 1\,, \;\; n_\phi = 0\,, \quad \xi = 0.1\,.
\end{aligned}
$$

so that $S_0 = Z_0 = 10$, $W_0(S_0, Z_0) = 10^{-11}$

	$\langle X\rangle$	$\Delta\langle X\rangle$	F^X	ΔF^X
S	$10 + 2\cdot 10^{-13}$	$2\cdot 10^{-14}$	$-9.9\cdot 10^{-28}$	–
Z	$10 + 2\cdot 10^{-13}$	$2\cdot 10^{-14}$	$5.6\cdot 10^{-28}$	–
T	31.4	$1.1\cdot 10^{-15}$	$-3.4\cdot 10^{-15}$	$-1.4\cdot 10^{-16}$
ϕ	0.15	$-1.3\cdot 10^{-15}$	$1.4\cdot 10^{-16}$	$-7.4\cdot 10^{-14}$
χ	0.05	$-7.6\cdot 10^{-14}$	$7.5\cdot 10^{-15}$	$1.5\cdot 10^{-14}$

$$
\begin{aligned}
m_{3/2}^2 &= 9.5\cdot 10^{-31}\,, \quad \Delta m_{3/2}^2 = -5.1\cdot 10^{-14}\,, \\
D_X &= 3.7\cdot 10^{-27}\,, \quad \Delta D_X = 3.8\cdot 10^{-14} \\
V_0 &= 1.2\cdot 10^{-32}\,, \quad \Delta V_0 = 8.2\cdot 10^{-12}
\end{aligned}
$$

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2) $W_0 \ll 1$

3) One does not consider too high operators in K or W

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1) $W_0 \ll 1$ in KKLT-like models more important than previously thought. Crucial to consistently freeze heavy moduli and have a mass hierarchy between light and heavy fields

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2 relevant applications of our results:

1) $W_0 \ll 1$ in KKLT-like models more important than previously thought. Crucial to consistently freeze heavy moduli and have a mass hierarchy between light and heavy fields

2) Obstruction due to gauge invariance (nothing to do with gravity) in local string models with non-neutral moduli, where the latter are all neglected

The end