Some things are just not possible: CFT lessons for BSM phenomenology

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(SNS and INFN, Pisa)

Based on work with **Francesco Caracciolo, Riccardo Rattazzi, Erik Tonni, Alessandro Vichi**
0807.0004, 0905.2211 and work in progress

Our new toys

- AdS/CFT $\leftrightarrow$ RS
- Large extra dims
- Little Higgs
- Unparticles

Why CFT?

Conformal symmetry can be emergent in the IR

Scale invariance
(RG fixed point)

↓ assuming *Unitarity*

Conformal Invariance

Folk theorem, no known counterexamples

Polchinski ‘88

E.g.

$SU(N_c)$ gauge theories with N_f flavors have large "conformal windows"

Evidence from:

- large N Belavin, Migdal `74, Banks, Zaks `82
- SUSY Seiberg '94
- Lattice QCD ($N_c = 3$; $N_f = 12$)

  Appelquist et al, Deuzeman et al '08'09

AdS/CFT vs general case

In AdS/CFT, operator dimensions factorize:

$$[\Phi_1] = \Delta_1, \;\; [\Phi_2] = \Delta_2$$

$$\Downarrow$$

$$\Phi_1 \times \Phi_2 \supset \Phi_1 \Phi_2$$

$$[\Phi_1 \Phi_2] = \Delta_1 + \Delta_2 + O(1/N)$$

"multiparticle states"

More factorization examples

- any large N theory

- SUSY, for chiral primaries

$$\Delta \propto \boldsymbol{R}$$

No factorization in general

Wilson-Fischer $\lambda\varphi^4$ fixed point in $4-\varepsilon$ dimensions:

$$\gamma(\varphi^2) = \frac{\varepsilon}{3} >> \gamma(\varphi) = \frac{\varepsilon^2}{128}$$

No factorization in general

Wilson-Fischer $\lambda\varphi^4$ fixed point in 4–ε dimensions:

$$\gamma(\varphi^2) = \frac{\varepsilon}{3} >> \gamma(\varphi) = \frac{\varepsilon^2}{128}$$

2-D Ising model :

spin operator σ
energy density ε

$$\sigma \times \sigma = 1 + \varepsilon$$

$$[\sigma] = \frac{1}{8}, \ \ [\varepsilon] = 1$$

Let's do something useful *now*:

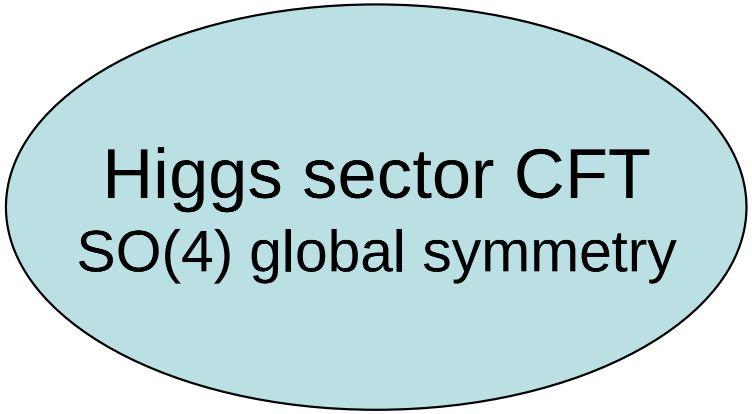
Non-factorization
for phenomenology

- Conformal technicolor

- $B\mu$ conformal sequestering

Conformal Technicolor

Luty, Okui `04
Luty `08



Conformal Technicolor

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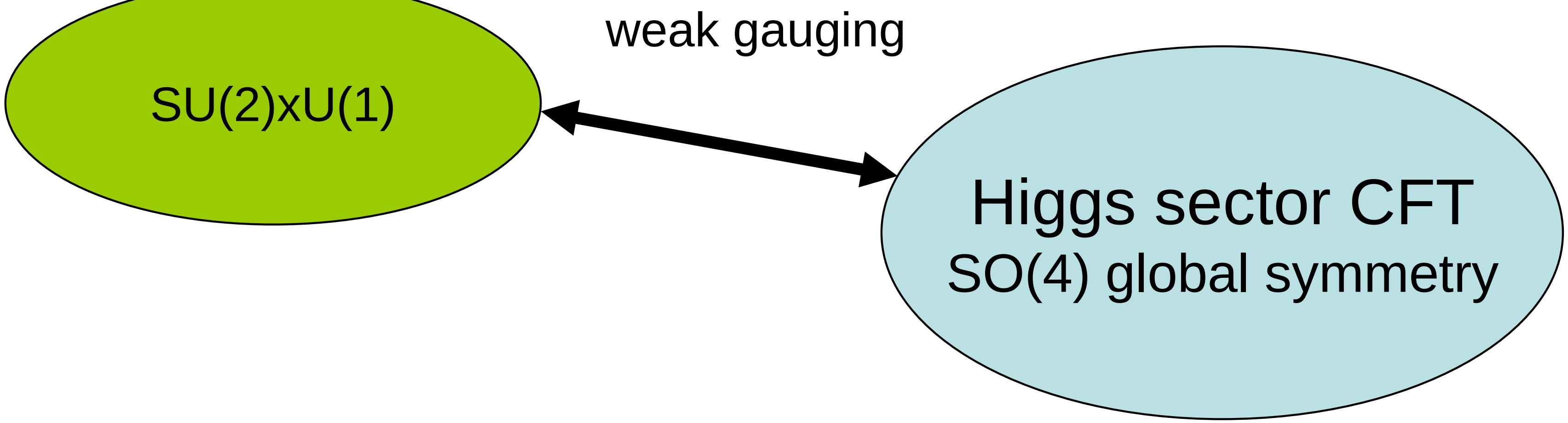

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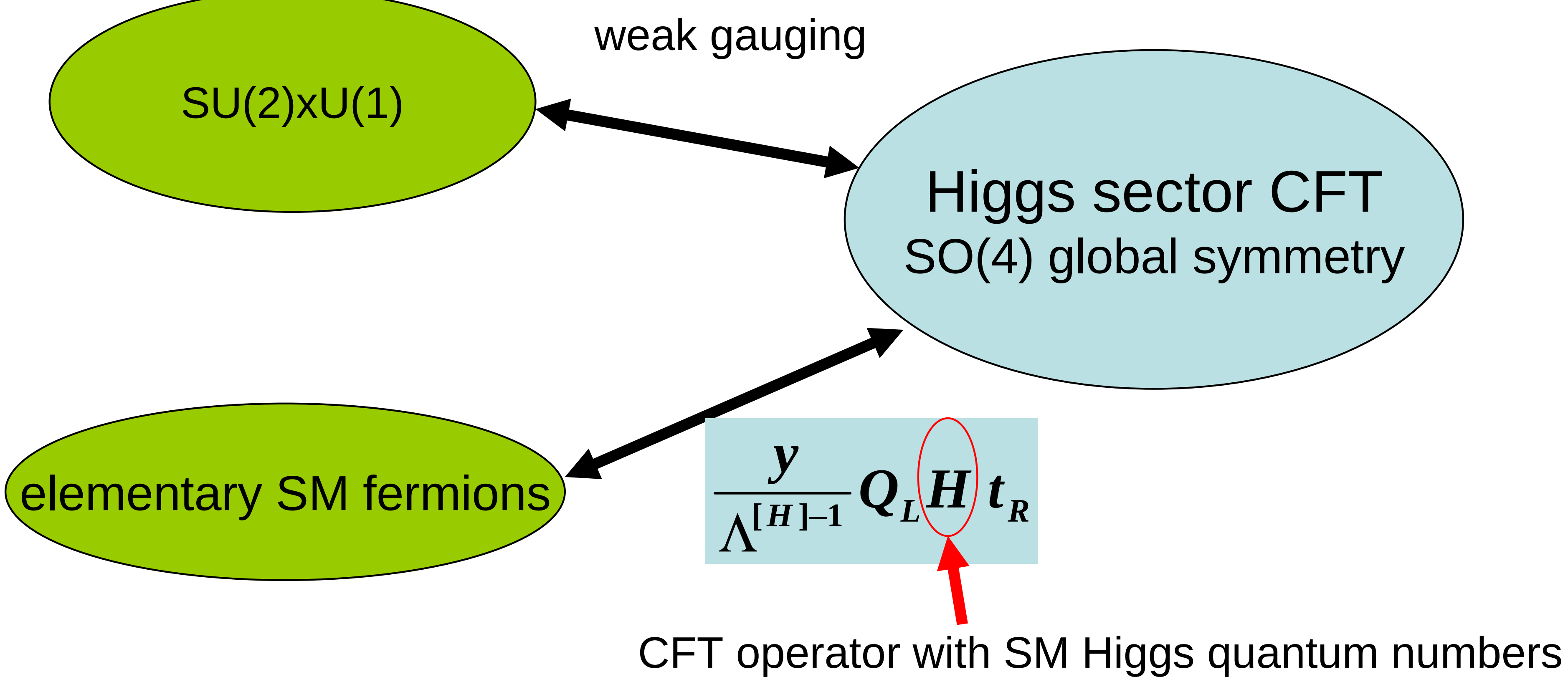

Conformal Technicolor

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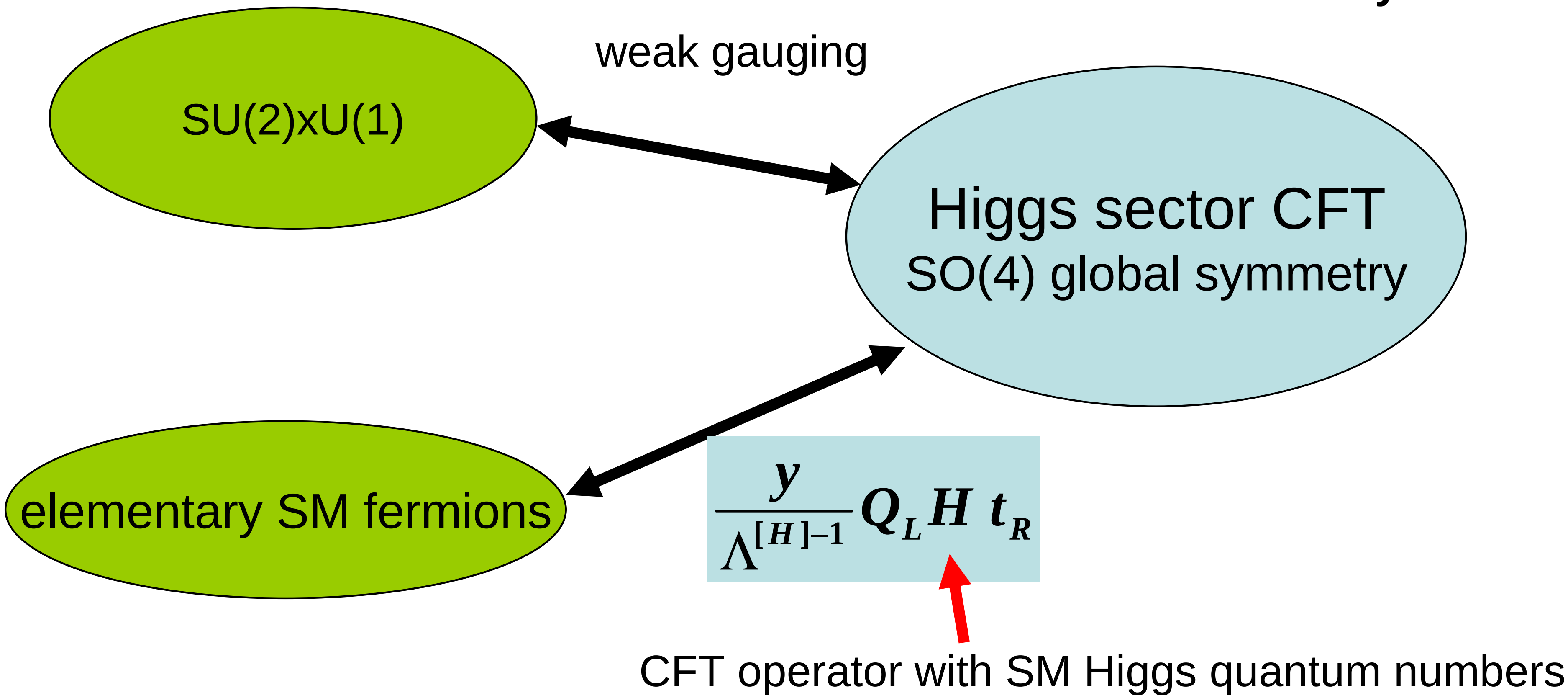

By assumption, conformal symmetry broken at the EW scale (explictly or radiatively by couplings to SM)

A dream theory for EWSB and flavor if anomalous dimensions jump

$$[H] \leq 1 + \frac{1}{few}$$

and

$$[H^+ H] \geq 4 - \frac{1}{few}$$

$$few \geq 3 \div 4$$

leading SO(4) **singlet** in the OPE

$$H^+ \times H = 1 + H^+ H + \dots$$

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No strongly relevant singlets,
Hierarchy solved up to **10^{few} TeV**

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$$H^+ \times H = 1 + H^+ H + \dots$$

Yukawas don't hit the strong scale until **10^{few} TeV**
FCNC sufficiently suppressed

No strongly relevant singlets,
Hierarchy solved up to **10^{few} TeV**

A dream theory for EWSB and flavor if anomalous dimensions jump

$$[H] \leq 1 + \frac{1}{few}$$ and $$[H^+ H] \geq 4 - \frac{1}{few}$$

$$few \geq 3 \div 4$$

(Compare to Unhiggs Stancato, Terning '08

$$[H] \approx 2, \quad [H^+ H] = 2[H] \approx 4$$

Hierarchy solved but flavor conservation not automatic)

Bμ conformal sequestering

$$\frac{1}{16\pi^2}\int d^4\theta \frac{X^\dagger}{M} H_u H_d \rightarrow \mu \int d^2\theta \, H_u H_d$$

$$\mu \sim \frac{F}{16\pi^2 M}$$

$$\frac{1}{16\pi^2}\int d^4\theta \frac{X X^\dagger}{M^2} H_u H_d \rightarrow B_\mu \, H_u H_d$$

$$B_\mu \sim \frac{F^2}{16\pi^2 M^2} \sim 16\pi^2 \mu^2$$

Gauge mediation μ/Bμ problem: **$16\pi^2$ too large**

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Gauge mediation μ/Bμ problem: **$16\pi^2$ too large**

Can be resolved if strong hidden sector dynamics suppresses **X^+X** w.r.t. **X**

Murayama, Nomura, Poland '07
Roy, Schmaltz '07

Requires $[X^+X] > 2[X] + O(1)$

A concrete CFT question motivated by these two examples

Is there a theoretical upper bound on $[\varphi^2]$ in terms of $[\varphi]$?

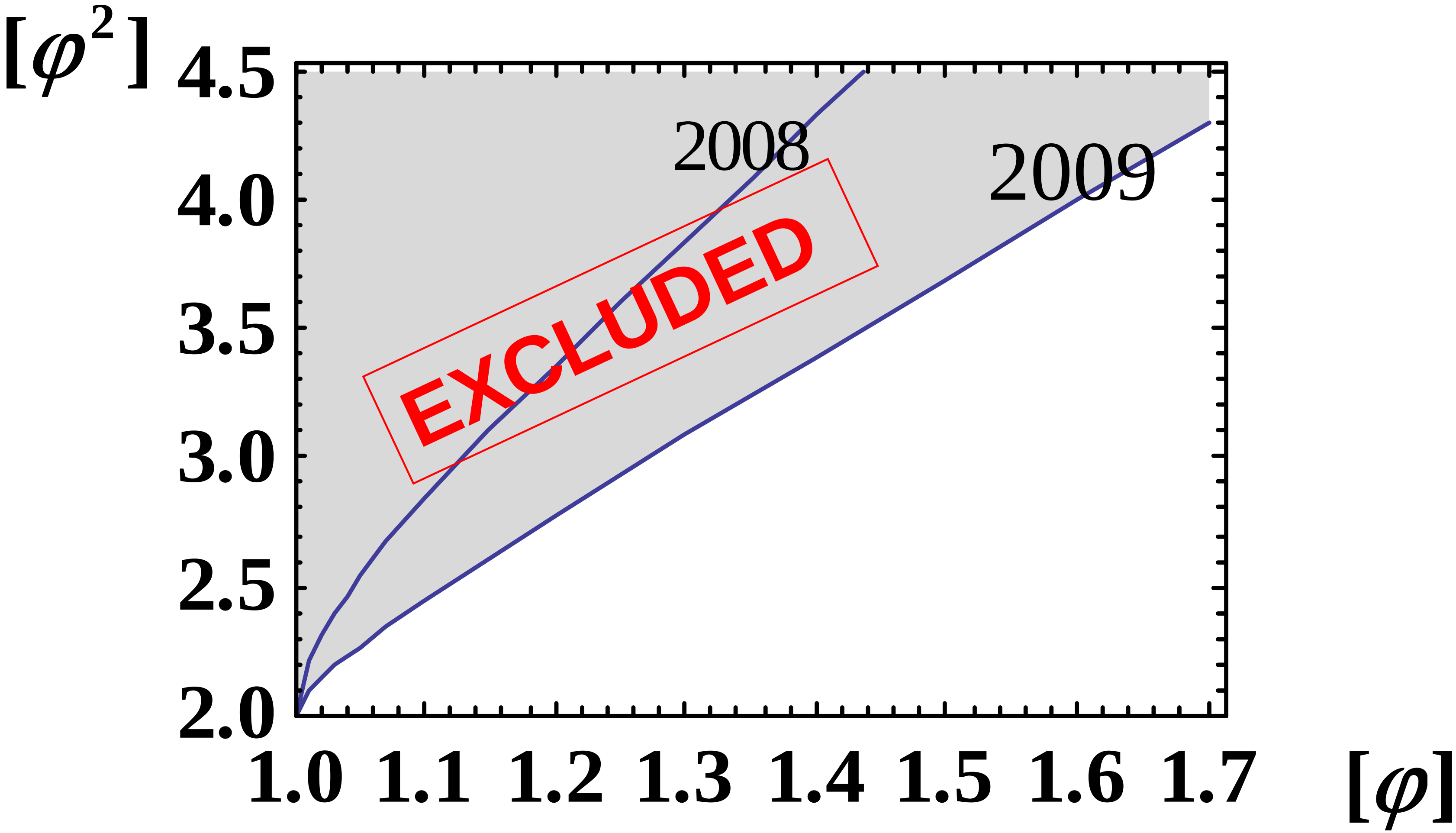
leading scalar in $\varphi\times\varphi$ OPE

Hermitian scalar

N.B.: **toy problem** since φ^2 not necessarily singlet under global symmetry (SO(4) for conformal TC, $U(1)_R$ for Bμ sequestering)

And the answer is

Rattazzi, S.R., Tonni, Vichi '08
S.R.,Vichi '09.



Universal theoretical upper bound:

$$[\varphi^2] \leq 2 + 0.7\sqrt{\gamma} + 2.1\gamma + 0.43\gamma^{3/2}, \ \gamma = [\varphi] - 1$$

And the answer is

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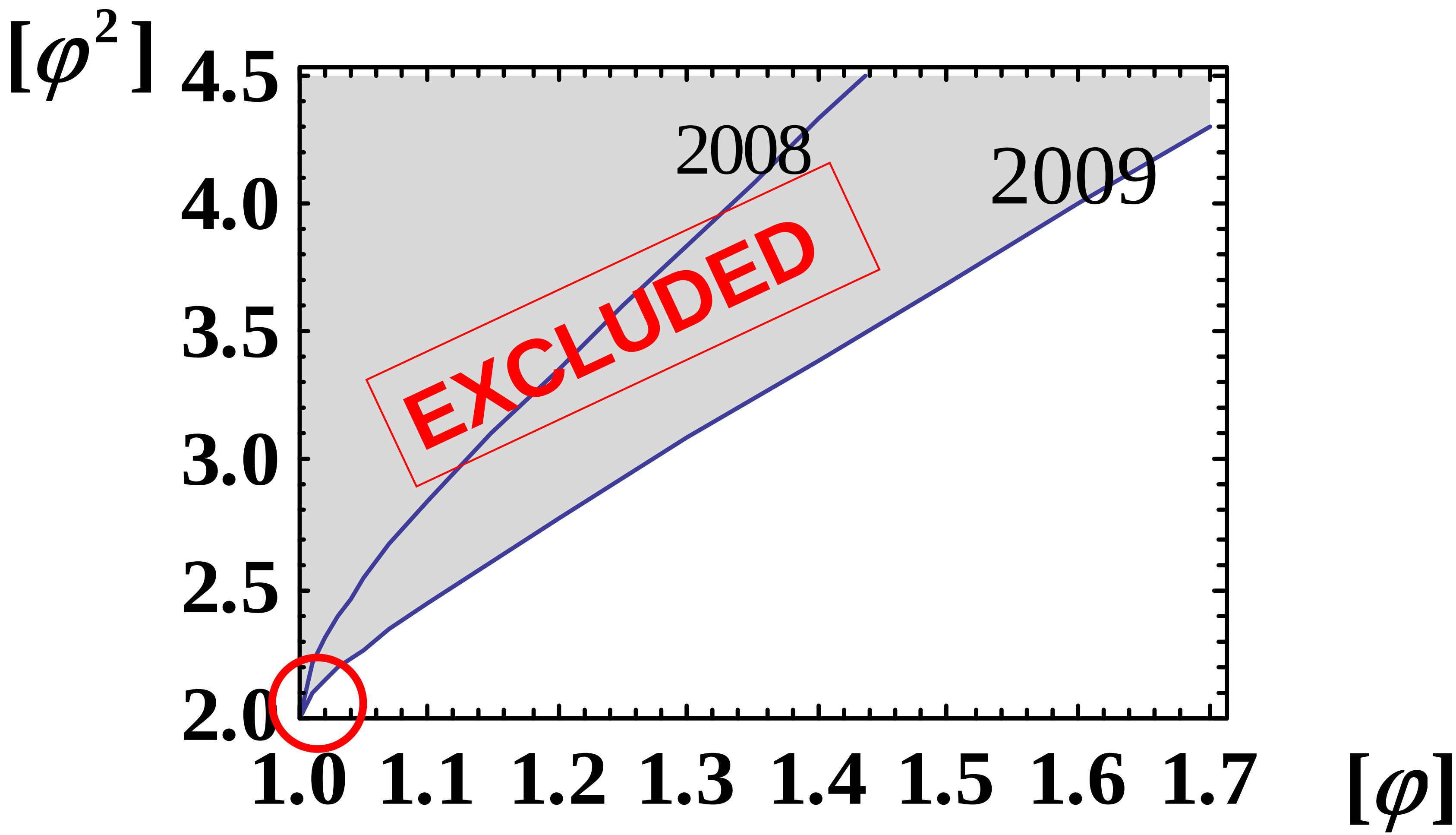

Continuous approach of free theory limit:

$$[\varphi^2]_{\max} \to 2 \quad \text{as} \quad [\varphi] \to 1$$

And the answer is

Rattazzi, S.R., Tonni, Vichi '08
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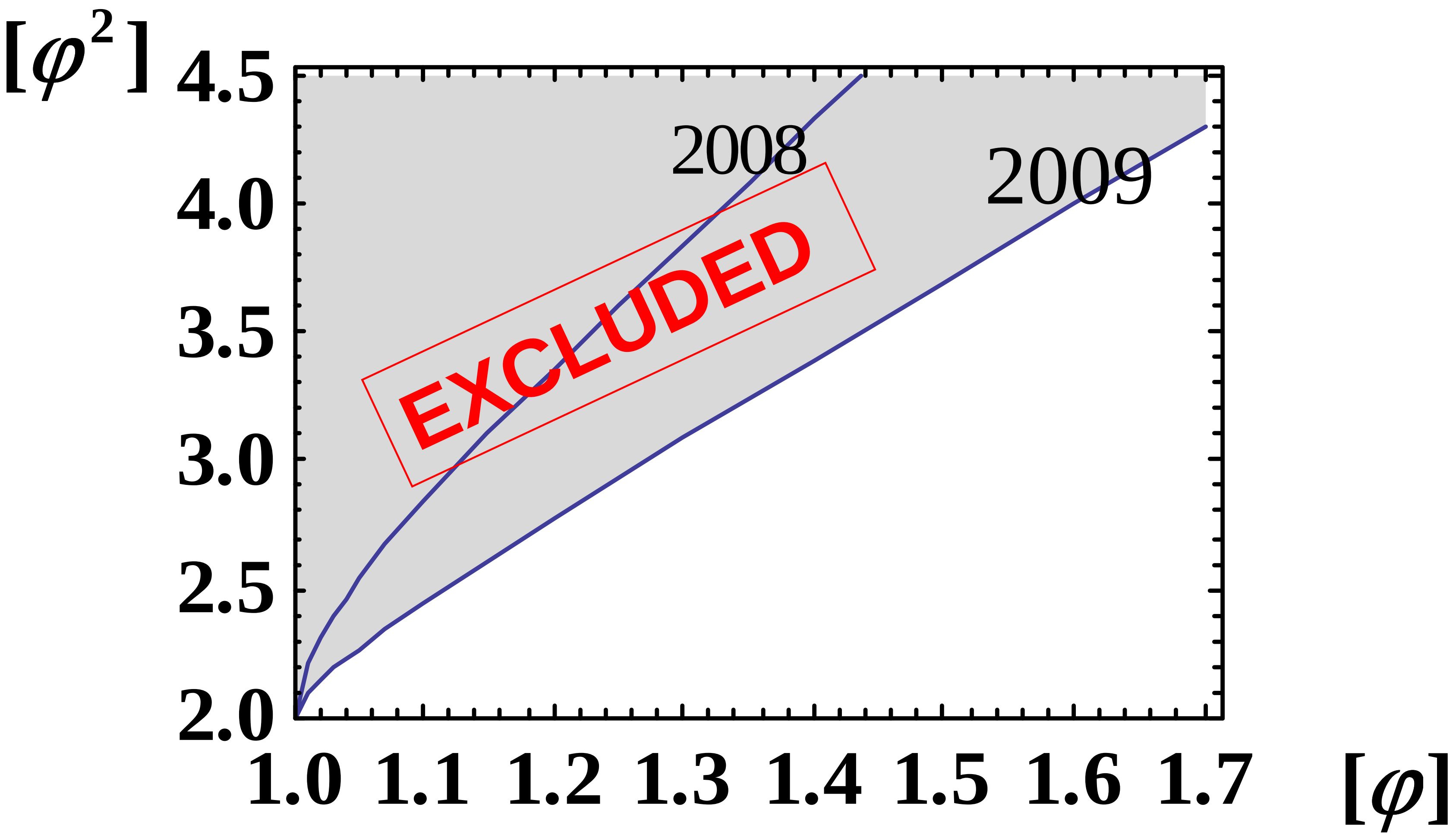

Bound interpolates **between 2 and 4** for $1<[\varphi]<1.7$

No doubt that the bound extends also for larger $[\varphi]$

And the answer is

Rattazzi, S.R., Tonni, Vichi '08
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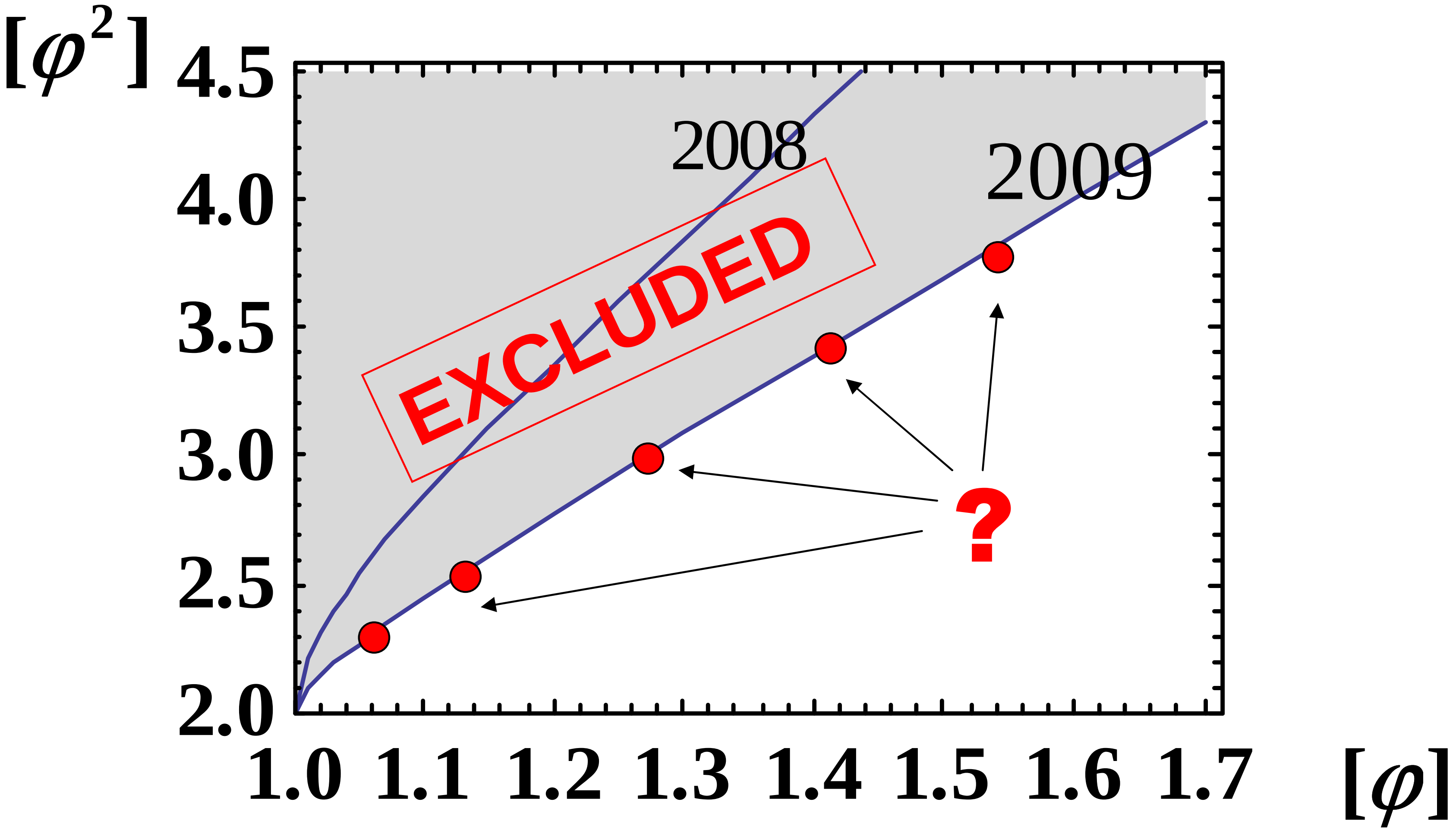
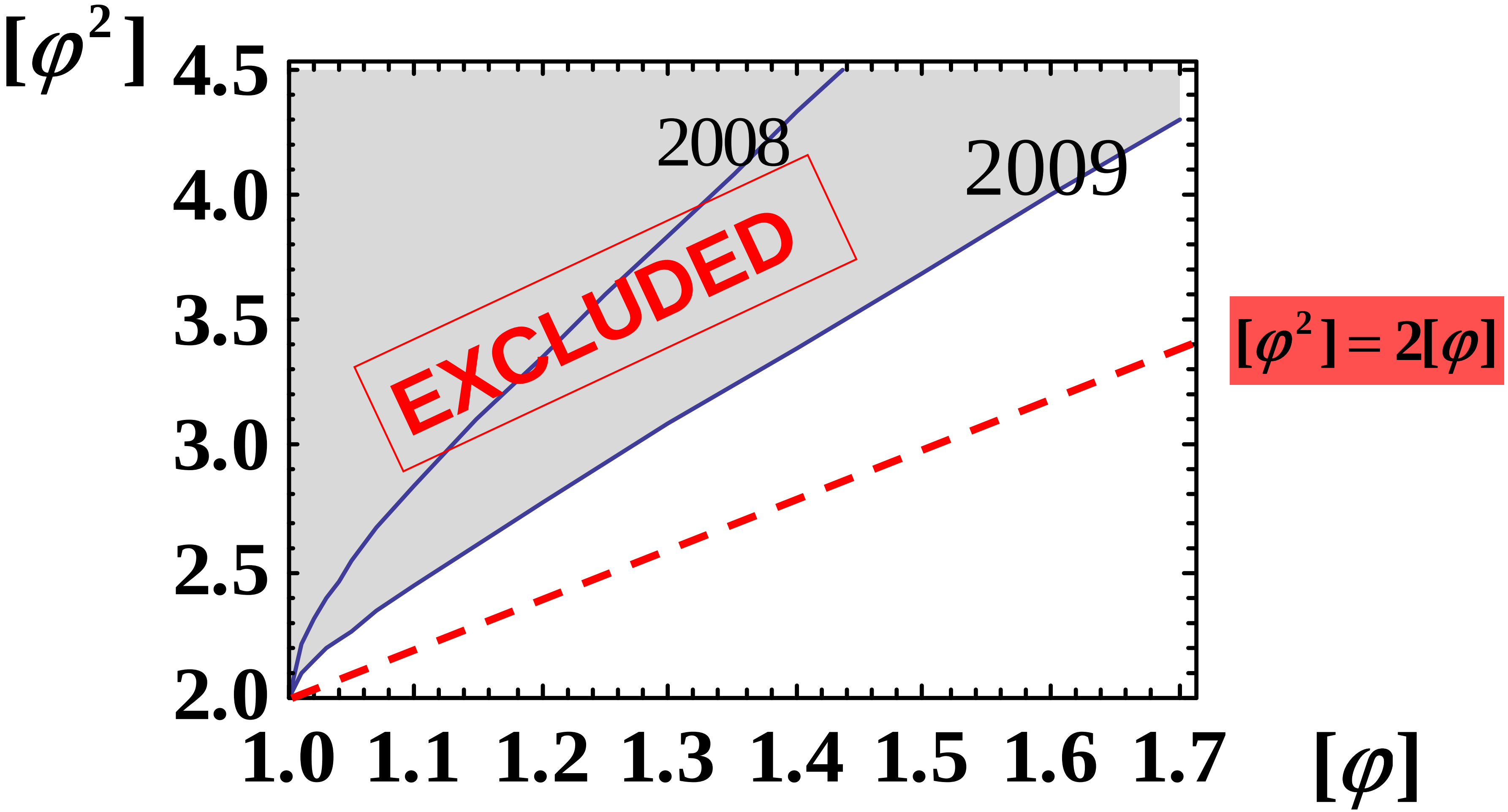


We don't know of any 4-D CFTs that saturate the bound,

but we presume they must exist

And the answer is

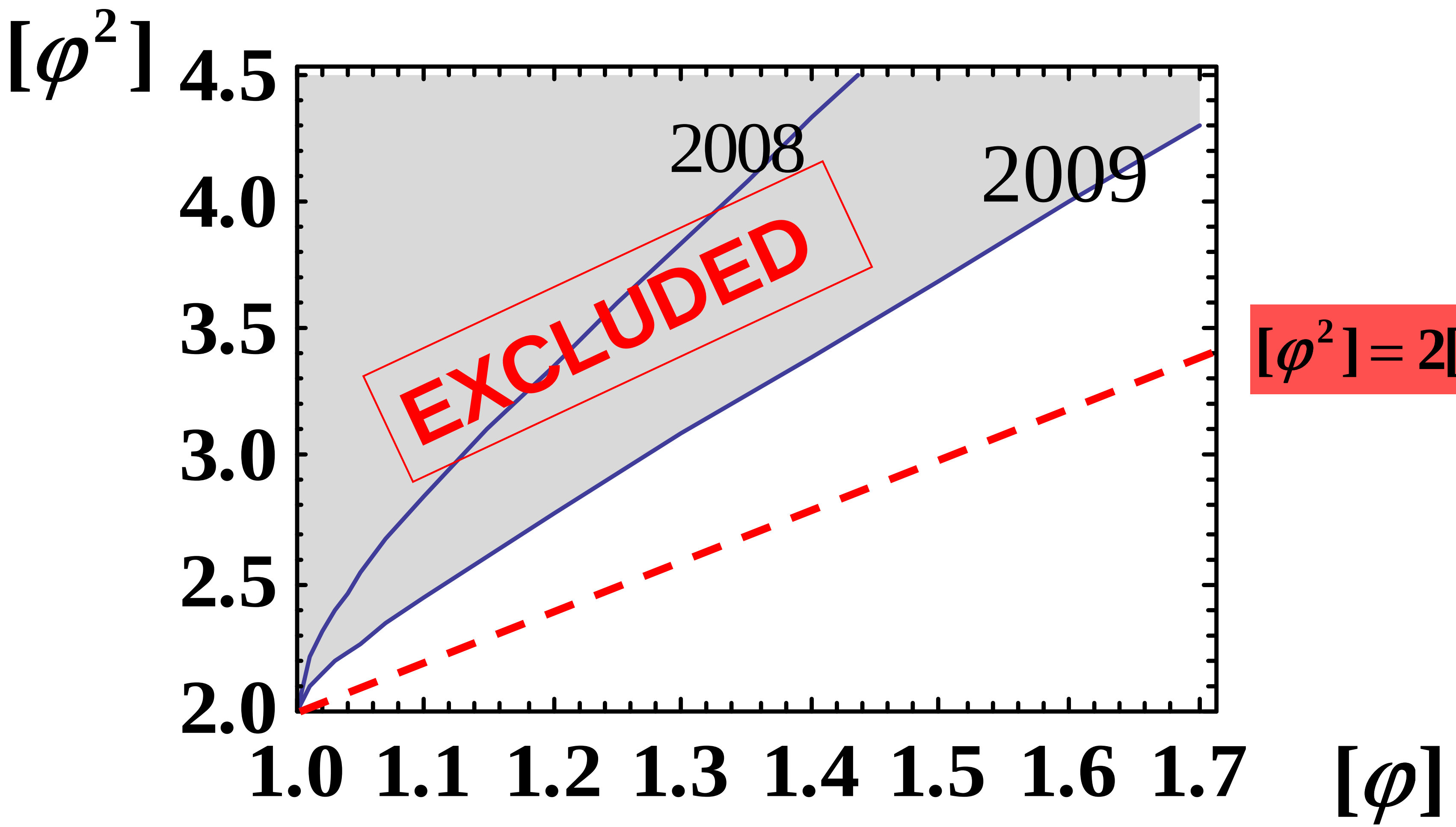
Rattazzi, S.R., Tonni, Vichi '08
S.R.,Vichi '09.



Bound is somewhat above the large N line of factorized dimensions

And the answer is

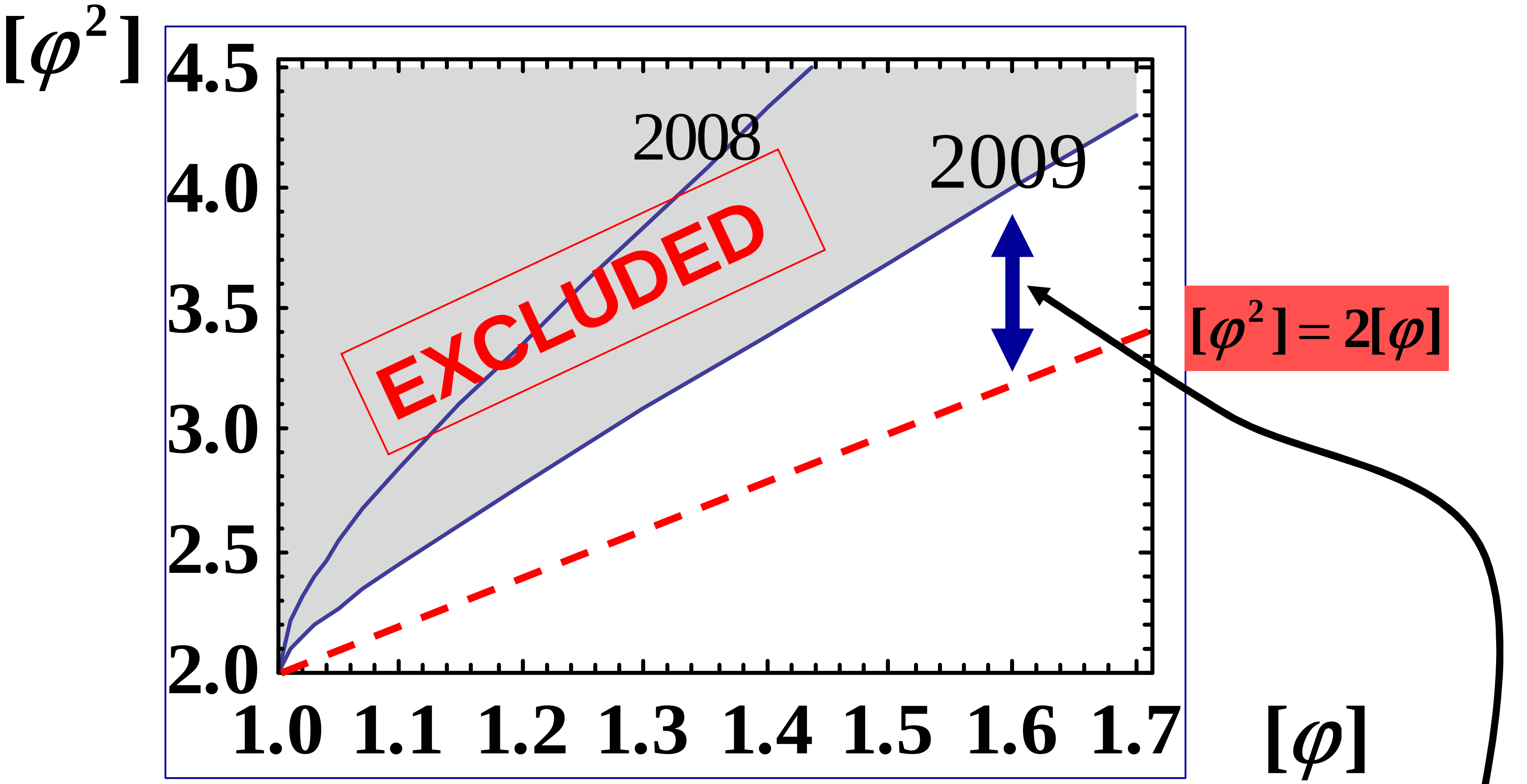
Rattazzi, S.R., Tonni, Vichi '08
S.R.,Vichi '09.



Still working on the bound for the lowest dim. **singlet,**
which is likely to be somewhat weaker

And the answer is

Rattazzi, S.R., Tonni, Vichi '08
S.R.,Vichi '09.



If this ***were*** the bound on the **singlet,** then…

Bμ conformal sequestering constrained but not excluded

And the answer is

Rattazzi, S.R., Tonni, Vichi '08
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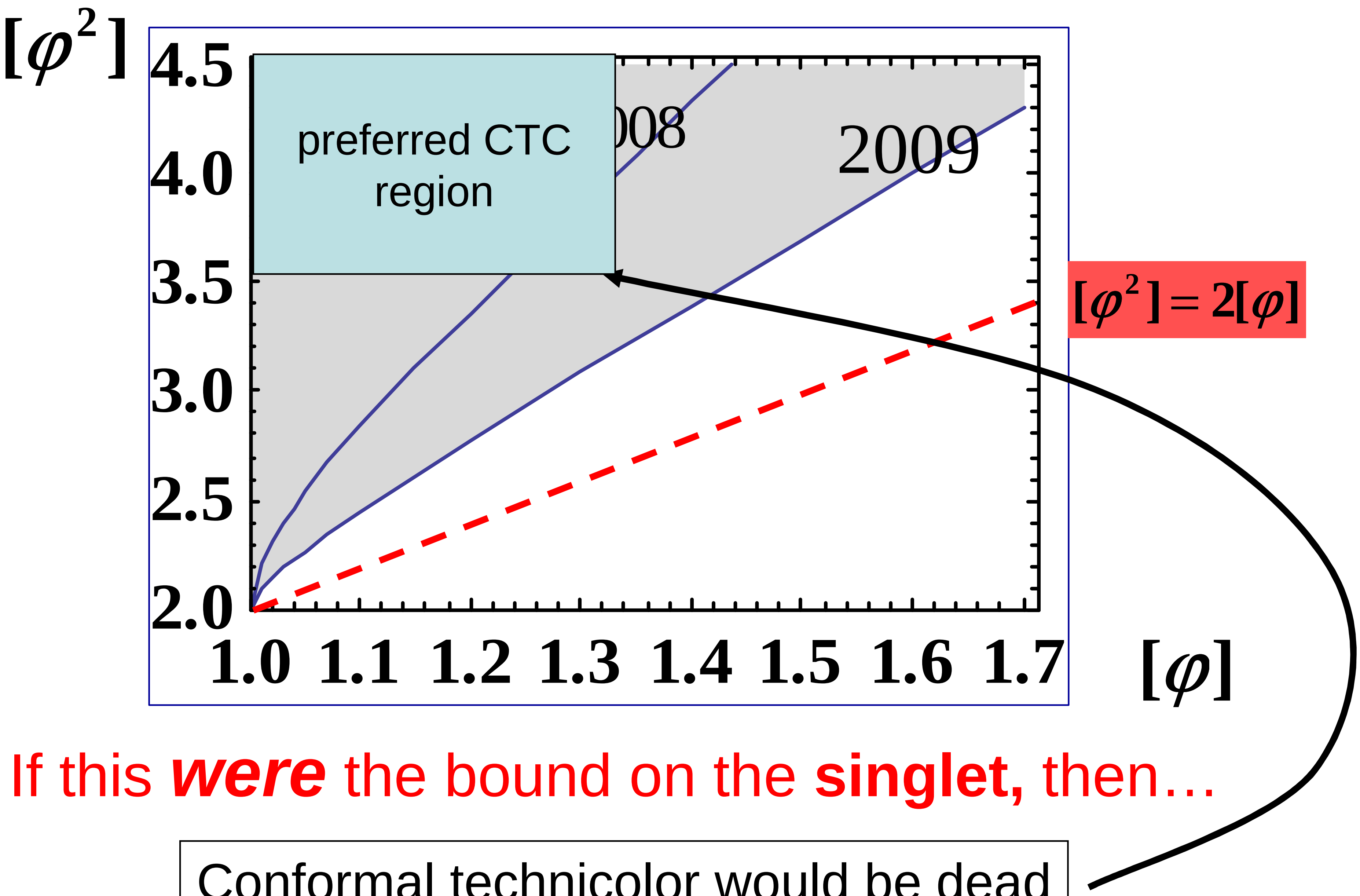


If this ***were*** the bound on the **singlet,** then…

Conformal technicolor would be dead

Back to crazy ideas: Unparticle self-interaction

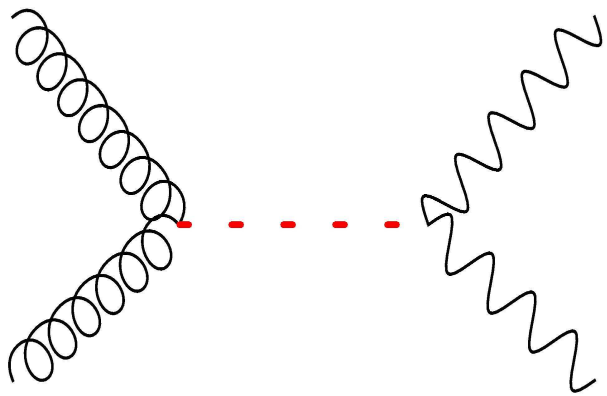
Feng, Rajaraman, Tu '08
Giorgi, Katz '09

Imagine unparticle sector coupled to the SM via

$$\frac{c_g}{\Lambda^d} O (G^a_{\mu\nu})^2 + \frac{c_\gamma}{\Lambda^d} O (B_{\mu\nu})^2$$

O – dimension *d* hidden scalar

Expect: $gg \to O \to \gamma\gamma$



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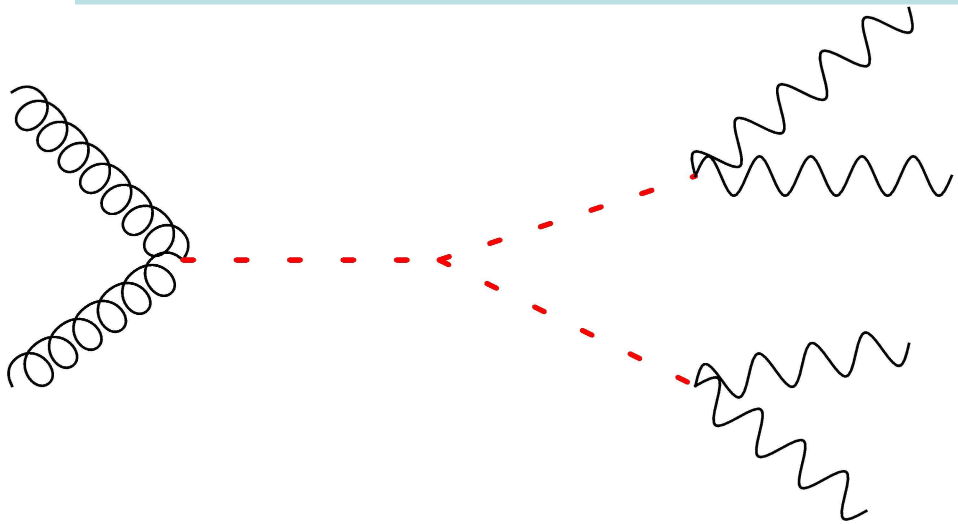
O – dimension d hidden scalar

Assuming $< OOO > \neq 0$

expect also $gg \to O \to OO \to 4\gamma$



The size of <OOO>?

Conformal symmetry fixes <OOO> up to a prefactor:

$$< OOO >= \frac{C_d}{|x-y|^d\,|x-z|^d\,|y-z|^d}$$

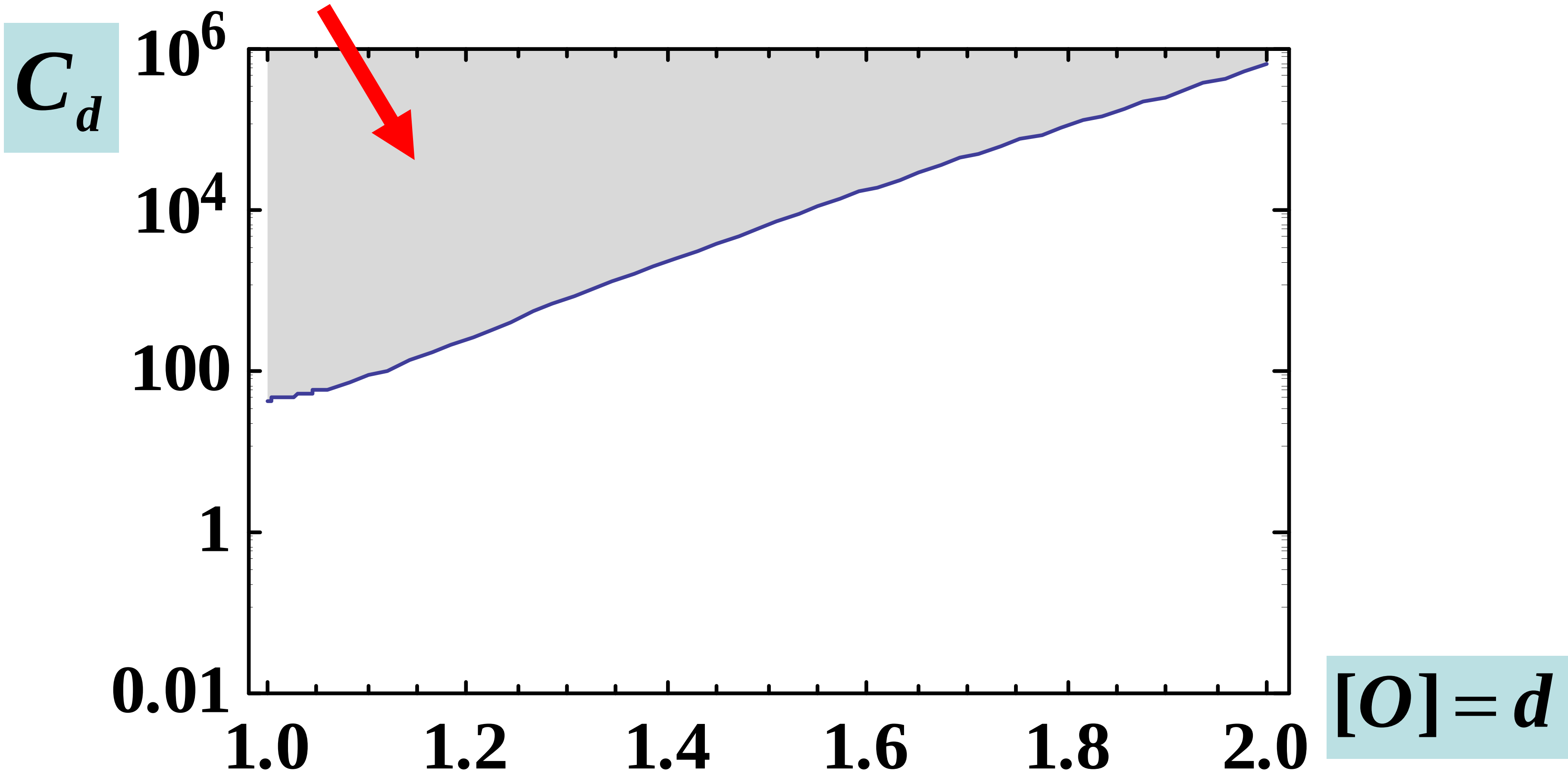
$$< OO >= \frac{1}{|x-y|^{2d}}$$

The prefactor C_d determines cross-section:

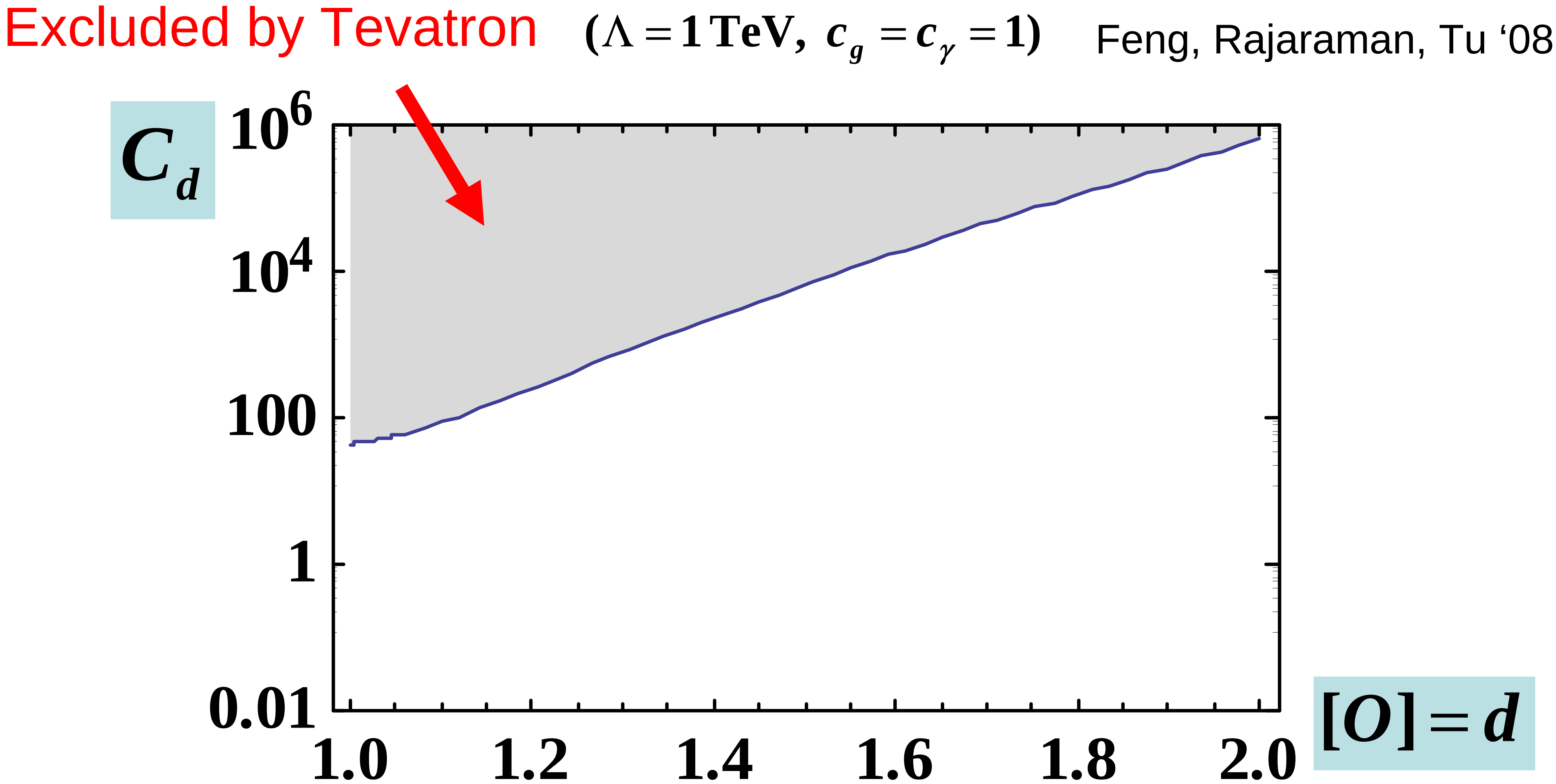
$$\sigma(gg \to 4\gamma) \propto C_d^2 \left(\frac{\hat{s}}{\Lambda^2}\right)^{3d} \frac{1}{\hat{s}}$$

Experimental bound on c_d

Excluded by Tevatron $(\Lambda = 1\,\text{TeV},\ c_g = c_\gamma = 1)$ Feng, Rajaraman, Tu ‘08

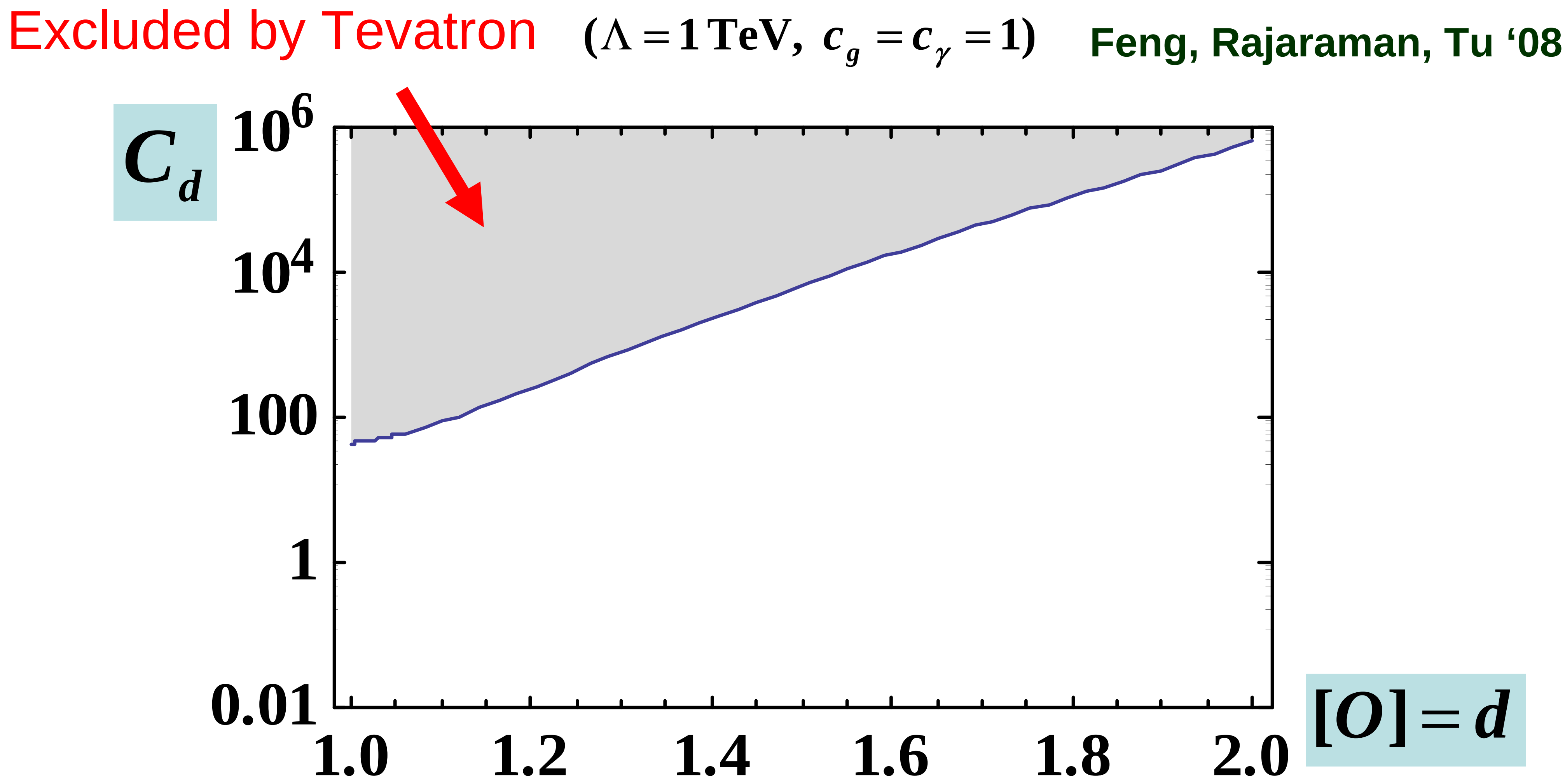


Experimental bound on C_d



If the true value of C_d anywhere near this bound, expect huge (**pb** to **nb**) LHC signal

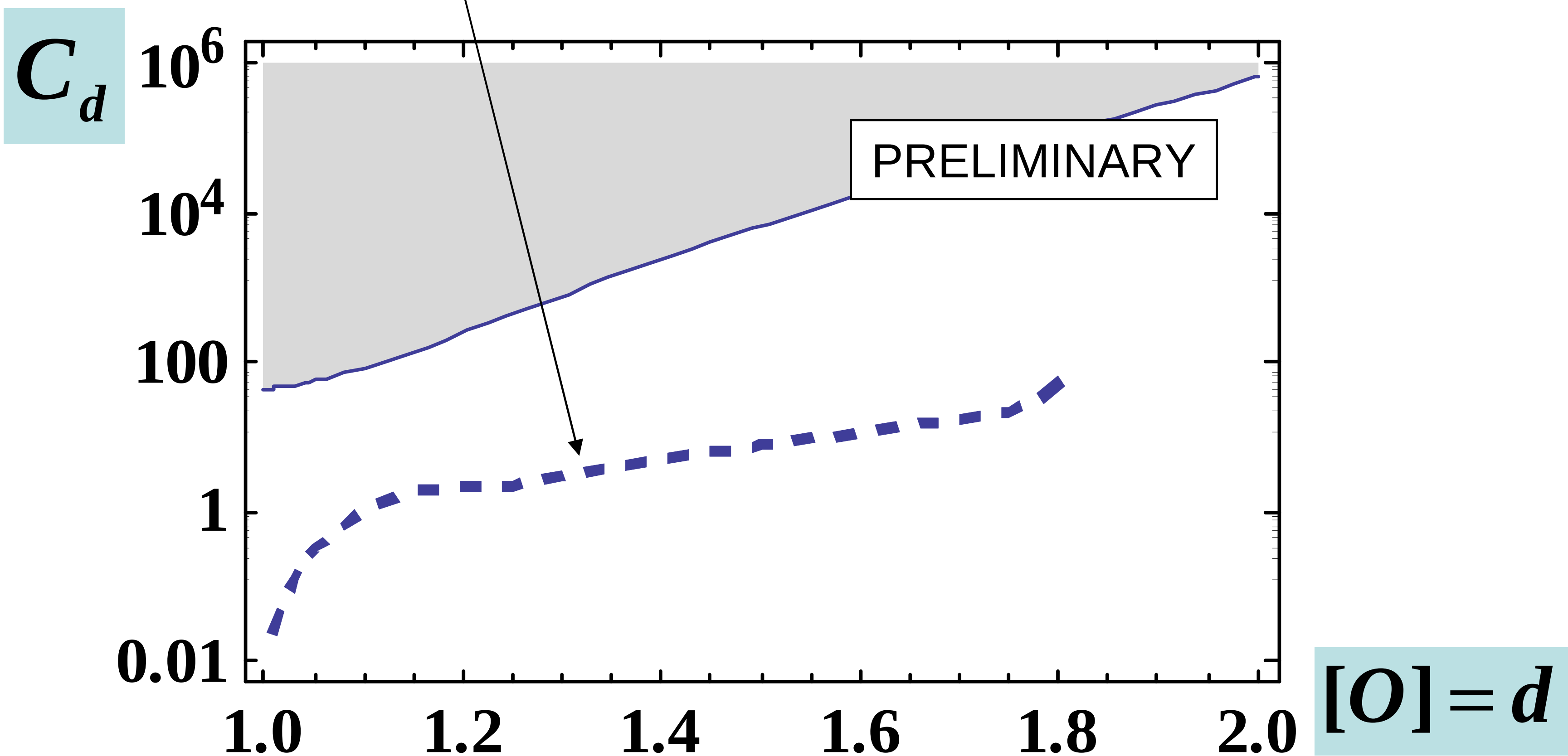
Experimental bound on c_d



But is it reasonable to have such large 3-point function?
NDA violated?

Theoretical bound on C_d

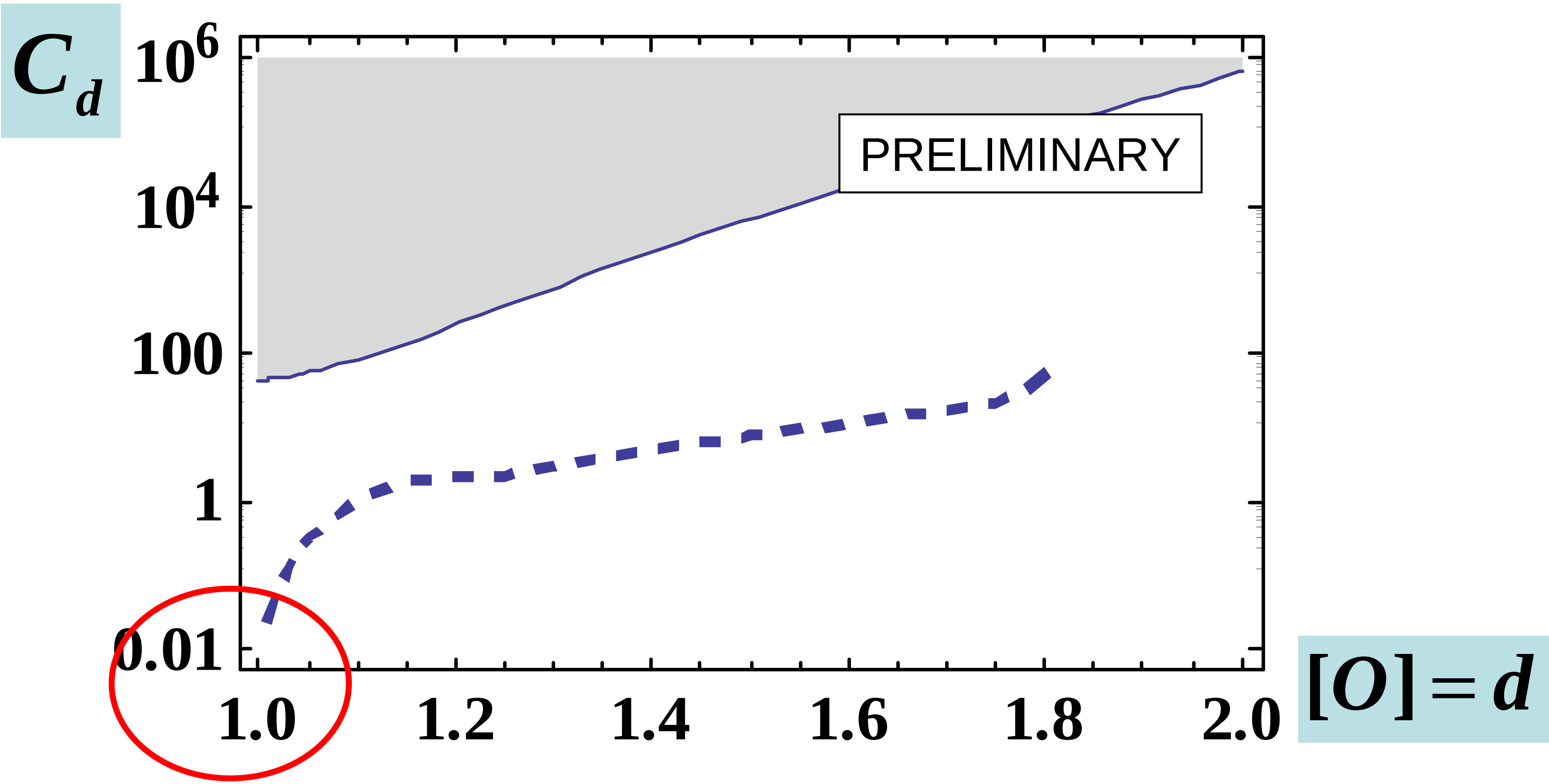
F. Caracciolo, S.R. (work in progress)



- Theoretical bound 2-3 orders of magnitude stronger
- Unparticle self-interactions hardly observable

Theoretical bound on C_d

F.Caracciolo, S.R. (work in progress)



Bound goes to zero as ***d*→1** - no cubic coupling in free theory

How do we do that

Method: **OPE** + crossing symmetry

$$\langle \Phi(x_1)\Phi(x_2)\Phi(x_3)\Phi(x_4)\rangle = \frac{g(u,v)}{|x_1-x_2|^{2d}|x_3-x_4|^{2d}}$$

u,v - conf. inv. cross-ratios

Method: **OPE** + crossing symmetry

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u,v - conf. inv. cross-ratios

$$g(u,v) = 1 + \sum_{\Delta,l}\left(C_{\Delta,l}\right)^2 G_{\Delta,l}(u,v)$$

Explicitly known functions (conformal blocks)

Method: **OPE** + crossing symmetry

$$\left\langle \Phi(x_1)\Phi(x_2)\Phi(x_3)\Phi(x_4)\right\rangle = \frac{g(u,v)}{|x_1-x_2|^{2d}|x_3-x_4|^{2d}}$$

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$$g(u,v) = 1 + \sum_{\Delta,l} \left(C_{\Delta,l}\right)^2 G_{\Delta,l}(u,v)$$

Coefficients of operators in $\Phi\times\Phi$ OPE

$$\Phi(x)\times\Phi(0) = 1 + \sum_{\Delta,l} C_{\Delta,l} O_{\Delta,l}(0)$$

Method: OPE + **CROSSING**

Can apply OPE in s and t-channel $\Longrightarrow$ crossing constraint:

$$g(u,v) = \left(\frac{u}{v}\right)^d g(v,u)$$

Method: OPE + **CROSSING**

Can apply OPE in s and t-channel $\Longrightarrow$ crossing constraint:

$$g(u,v) = \left(\frac{u}{v}\right)^d g(v,u)$$

Can be rewritten as a crossing deficit equation:

$$u^d - v^d = \sum_{\Delta,l} \left(C_{\Delta,l}\right)^2 \left[v^d G_{\Delta,l}(u,v) - u^d G_{\Delta,l}(v,u)\right]$$

crossing deficit
from unit operator

It's not easy to balance the crossing deficit

$\Longrightarrow$ nontrivial constraints on the spectrum and OPE coefficients

Conclusions

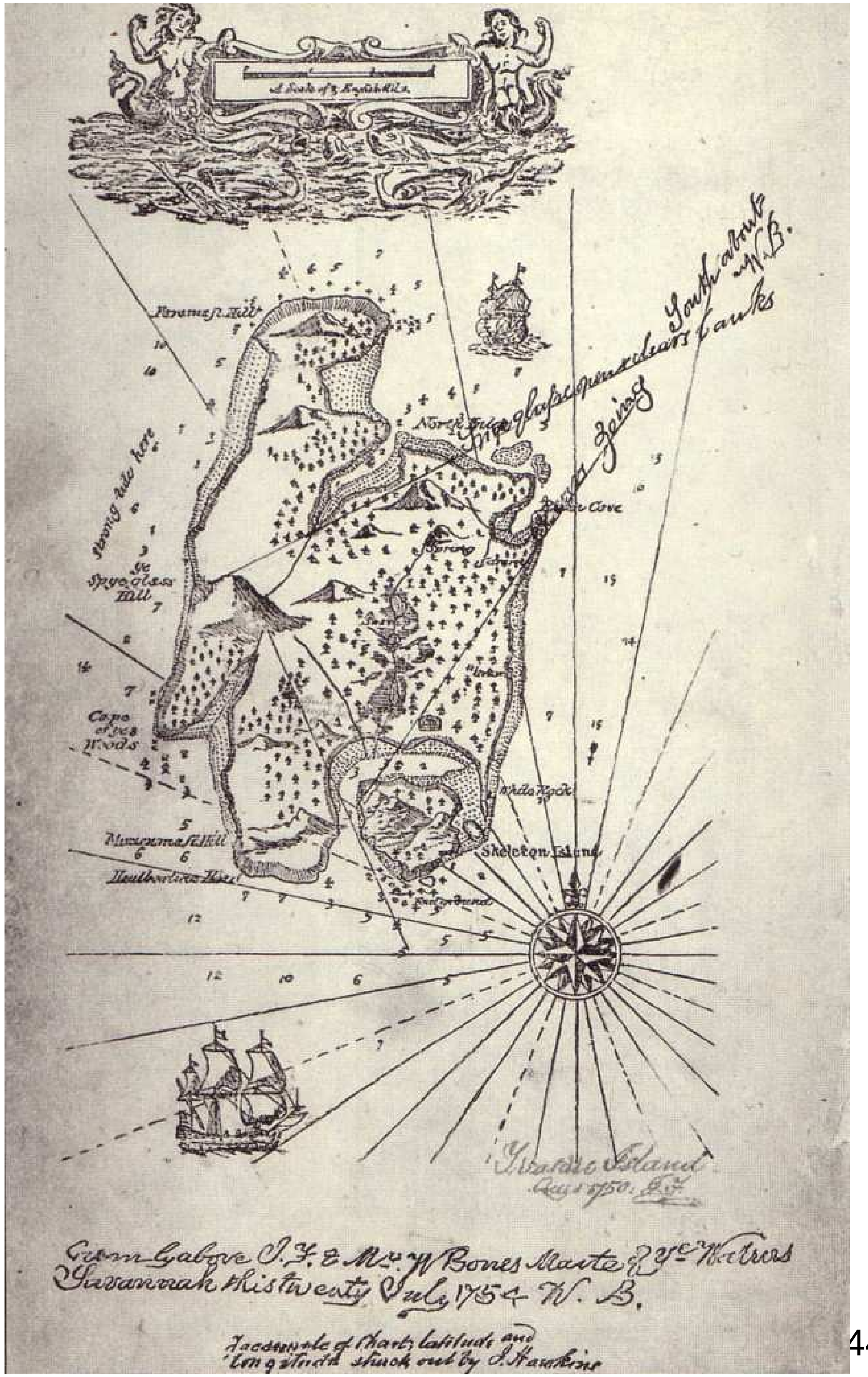
1. Phenomenological motivations $\Longrightarrow$ **need CFTs** which deviate from large N factorization of operator dimensions (i.e. without AdS duals)
2. To probe this phenomenon we derive **rigorous bounds** on how much the anomalous dimensions may jump
3. Similar methods $\Longrightarrow$ **bounds for OPE coefficients** (NDA made rigorous)

If we ever land again

on the treasure island

of strong coupling physics

it's good to know its borders



BACKUP

OPE

Primaries

Descendants (fixed by symmetry)

$$\phi(x)\phi(0) \sim \frac{1}{|x|^{2d}} \left\{ \mathbb{1} + \sum_{l=2n} c_{\Delta,l} \left[|x|^{-\Delta} K_l(x) \cdot O_{\Delta,l}(0) + \cdots \right] \right\}$$

$d \equiv [\varphi]$

by Bose symmetry

$$K_l(x) = \frac{x^{\mu_1} \cdots x^{\mu_l}}{|x|^l}$$

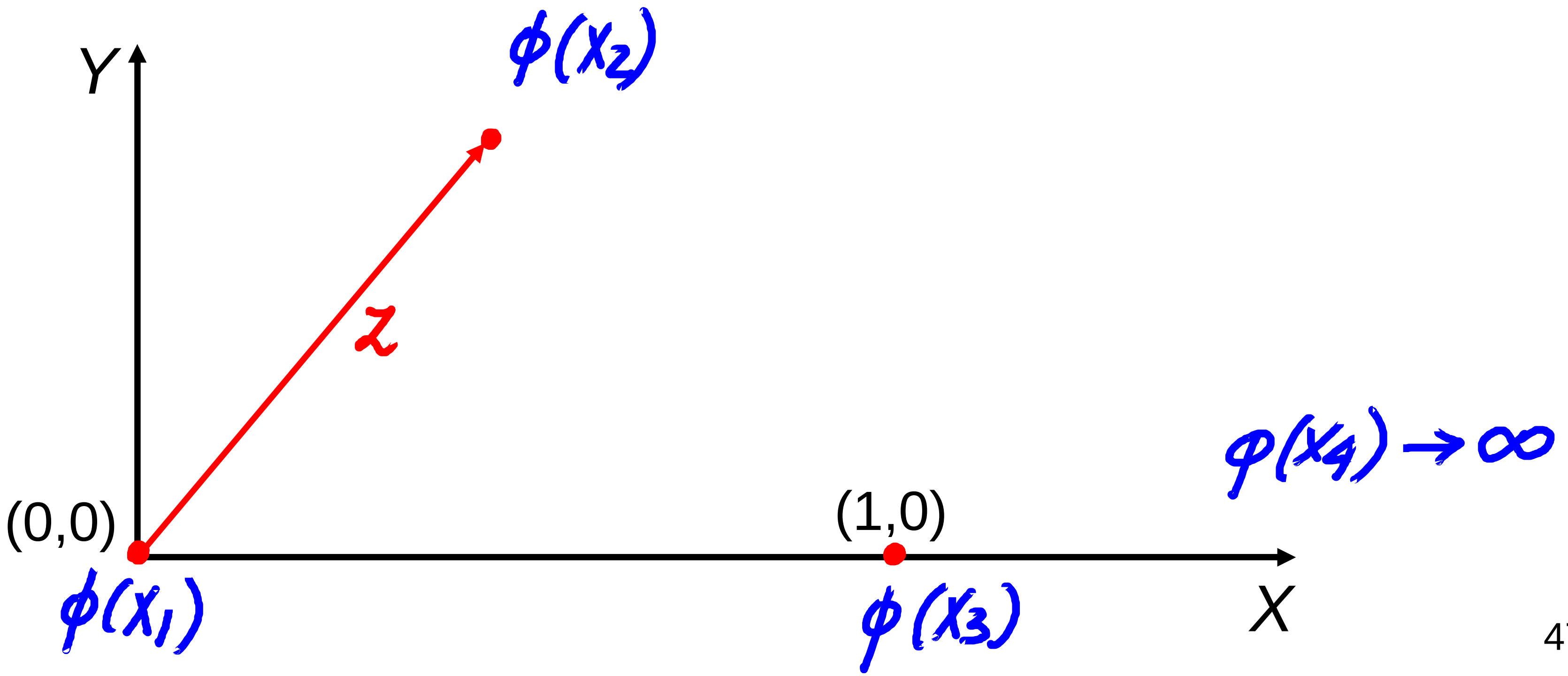
$\Delta \geq 1$ (l=0)
$\Delta \geq l+2$ (l=2,4,6…)
Unitarity bounds
Mack'77

4D Conformal Blocks in closed form [Dolan, Osborn, 2001]
It makes you feel powerful!

$$g_{\Delta,l}(u,v) = \frac{z\bar{z}}{z-\bar{z}}[f_{\Delta+l}(z)f_{\Delta-l-2}(\bar{z}) - (z \leftrightarrow \bar{z})]$$

$$f_\beta(z) = z^{\beta/2}\,{}_2F_1(\frac{\beta}{2},\frac{\beta}{2},\beta;z)$$

$$u = z\bar{z}, \quad v = (1-z)(1-\bar{z})$$



2D and 3D examples

show that $\gamma_{\phi^2} >> \gamma_\phi$ is not impossible.

Ising model: $\sigma \times \sigma = \mathbf{1} + \varepsilon$

2-dimensions (Onsager)	$[\sigma] = \mathbf{1/8}, \quad [\varepsilon] = \mathbf{1}$
3-dimensions (ϵ- and high-T expansions, Monte-Carlo)	$\gamma_\sigma \approx \mathbf{0.02}, \quad \gamma_\varepsilon \approx \mathbf{0.4}$

Extending analysis to 3d?

difficulty: finding 3d conformal blocks

(in odd dim's conformal blocks do not factorize as f(z)f(zbar))

Non-trivial extension for globally-symmetric case?

$$\phi_a \times \phi_b = \delta_{ab}(1 + O^{(1)}) + O^{(2)}{}_{ab} + \ldots$$

$$\ldots \supset J^{\mu}{}_{ab}$$

-two inequivalent crossing-symmetric 4-pt functions:

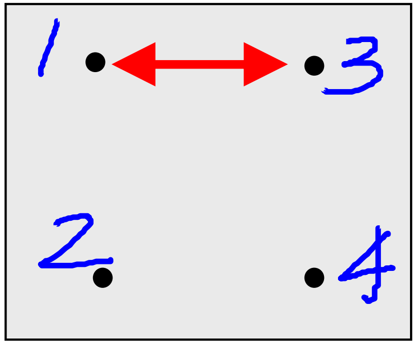
$$\langle \phi_1\phi_1\phi_1\phi_1 \rangle \quad \langle \phi_1\phi_2\phi_1\phi_2 \rangle$$

-OPE contains singlets and symmetric-traceless tensors (*even spin*);
antisymmetric tensors (*odd spin*)

Can one bound $[O^{(1)}]$ in a model-independent way?

Crossing Symmetry –
can we balance the budget deficit?

$$v^d g(u,v) = u^d g(v,u)$$



crossing deficit
from unit operator

$$u^d - v^d = \sum_{\Delta,l} \left(\lambda_{\Delta,l}\right)^2 \left[v^d g_{\Delta,l}(u,v) - u^d g_{\Delta,l}(v,u)\right]$$

Sum Rule:

$$1 = \sum_{\Delta,l} \lambda_{\Delta,l}^2 F_{d,\Delta,l}(u,v)$$

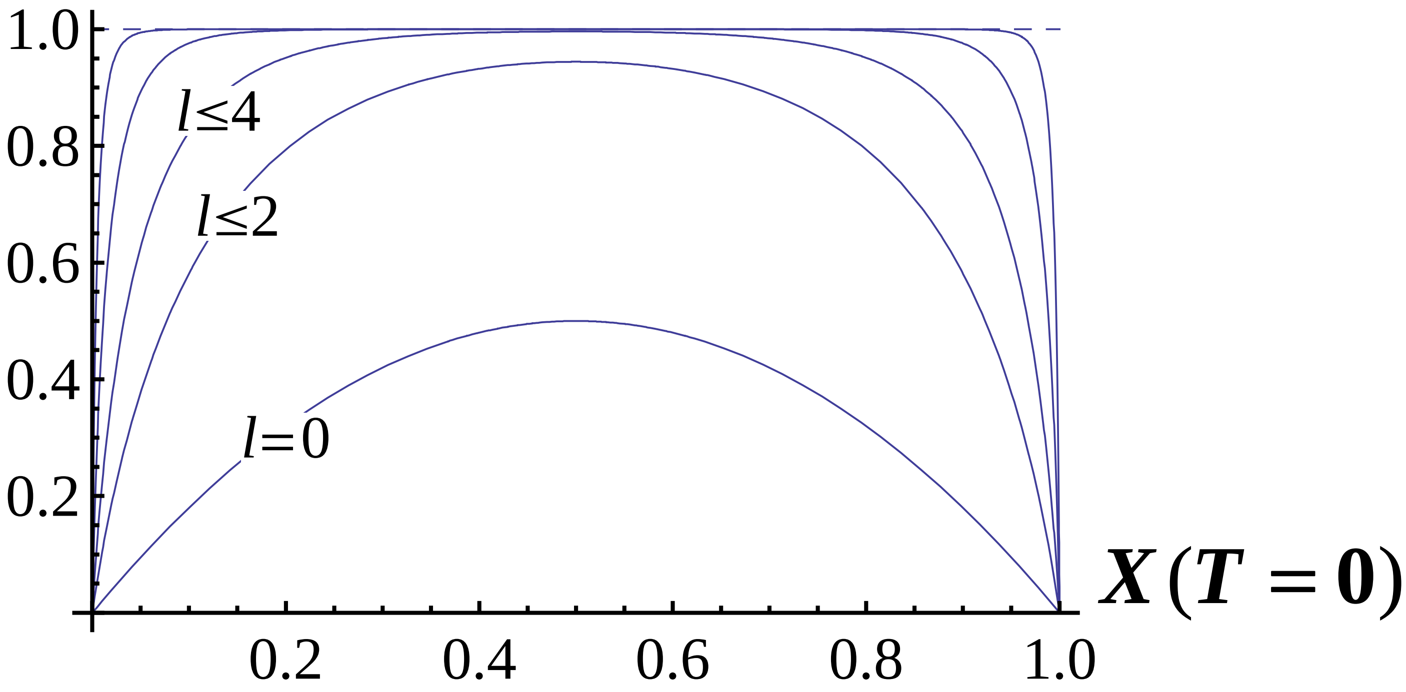
$$F_{d,\Delta,l}(u,v) := \frac{v^d g_{\Delta,l}(u,v) - u^d g_{\Delta,l}(v,u)}{u^d - v^d}$$

Sum rule convergence in free scalar theory

$$\phi \times \phi = \sum_{l=2n} \phi \overleftrightarrow{\partial}^{2n} \phi$$

twist 2 fields only

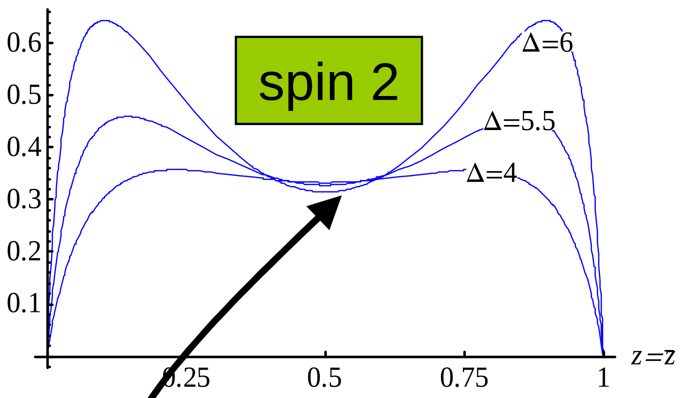
$$\lambda_l^2 = 2^{l+1} \frac{(l!)^2}{(2l!)^2}$$

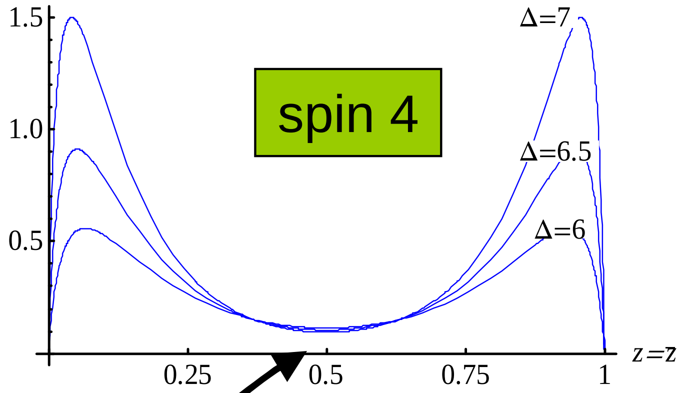


Monotonic convergence

How can this be at all possible?

Example:
d=1.01





- **Must sum up to ≡1 with positive coefficients**
- **F''(1/2)>0 for all higher spins and for all scalars with small Δ**
- **⟹ a scalar with small Δ must be present**