Introduction

Background: The WMAP results have shown the primordial fluctuations which seed structure formation to be predominantly gaussian, scalar and adiabatic. Such perturbations are characterised by the primordial power spectrum (PPS) $P_R(k)$. The data points $d_\ell$ of CMB anisotropy, galaxy clustering, Lyman $\alpha$ forest, cluster abundance or weak lensing measurements can be written as

$$d_\ell = \int_0^\infty K_\ell(k) P_R(k) \, dk + n_\ell,$$  \hfill (1)

where $n_\ell$ is noise and the kernel $K_\ell(k)$ depends on the late-time cosmological parameters.

Aim: To estimate $P_R(k)$ from the data, assuming $K_\ell(k)$ is known.

Motivation: The PPS is important both for cosmological parameter estimation and for distinguishing between models of inflation. Given the uncertainty in the physics behind inflation, a model-independent approach to estimating the PPS is essential.

Novelty: Most such attempts have used the WMAP TT data alone. By using several data sets we can recover $P_R(k)$ over a larger $k$ range with increased accuracy.

Problem: The convolution with $K_\ell(k)$ acts as a smoothing operation. Conversely, noise in the data is amplified in the reconstructed $P_R(k)$.

Solution: Regularisation techniques are used to control the error propagation at the cost of introducing a bias.

Tikhonov regularisation

Idea: Maximise the smoothness and the fit to the data of the recovered PPS.

Method: Discretising the integral in eq. (1) gives

$$d_\ell = \sum_i W_{\ell,i} s_i + n_\ell, \quad s_i \equiv P_R(k_i).$$

We take our estimate $\hat{s}$ of $s$ as the vector which minimises

$$Q(s) = -2 \ln P(d|s) + \lambda s^T L^T L s,$$

where $P(d|s)$ is the likelihood function and $L$ is a discrete approximation to the derivative operator. The first term on the lhs ensures the estimate matches the data while the second term penalises roughness. To characterise the error the Bayesian and frequentist covariance matrices $\Sigma_B \equiv (s - s)(s - s)^T$ and $\Sigma_F \equiv (s - s)(s - s)^T$ are used. To characterise the resolution we introduce the function $R(k, k')$ which satisfies

$$\langle \hat{P}_R(k) \rangle = \int_0^\infty R(k, k') P_R(k') \, dk'.$$

Conclusion

We have shown that Tikhonov regularisation and the Backus-Gilbert method produce high resolution reconstructions of the PPS from multiple noisy data sets. Both methods give consistent results and have well-defined error estimates. The recovered PPS show interesting features on large scales.