New Crab Nebula limits on Planck-scale suppressed LV in QED

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Motivations

Motivations:

- Lorentz Invariance Violation is found in several Quantum Gravity scenarios (tensor VEV, sp foam, LQG, NC geometry, condensed matter analogues of emergent gravity). In general one finds modified dispersion relations

Note: rotation invariance preserved

→ $$E^2 = p^2 + m^2 + \eta_1 \frac{\mu^2}{M} p + \eta_2 \frac{\mu}{M} p^2 + \sum_{n\geq 3} \eta_n \frac{p^n}{M^{n-2}} \qquad \left(p \equiv |\vec{p}|\right)$$

- The whole Lorentz Group is experimentally untestable (arbitrarily large boosts)
- **Need of high energy tests of Lorentz Invariance**

O(1) effects $\xrightarrow[QG]{}$ $E \sim M_{Pl}$ $\qquad E_{UHECR} \sim 10^{-8}\, M_{Pl}$ → **hopeless!!**

BUT there are **special situations** where tiny corrections can be magnified to sizable effects

Windows on Quantum Gravity

- **cumulative effects:** long baseline dispersion and vacuum birefringence (e.g. of signals from GRBs AGNs, pulsars)

} purely **kinematic effects** only the form of the MDR matters

- **anomalous** (normally forbidden) **threshold reactions** allowed by LV terms (photon decay, Vacuum Cherenkov emission…)
- **Shifting of existing threshold reactions** (e.g. photon annihilation onto FIR from blazars, GZK reactions)
- **LV induced decay** not characterized by a threshold (e.g. photon splitting)

} **Dynamical effects**
need to **compute reaction rates**, to rely on **energy-momentum conservation**, to retain almost all standard (particle) physics. Natural framework: **Effective Field Theory (Standard Model Estension)**

for further details see e.g. **D. Mattingly, 2005**

Beyond Lorentz Invariance

Our framework

Dimension 5 Standard Model Extension: include dimension 5 LV operators in the SM preserving gauge and rotation invariance and quadratic in the fields
Myers & Pospelov, 2003
Contribution at order **p^3/M** to the MDR
(neglect lower order terms, that have strong experimental bounds).

Modified Dispersion Relations

$$\omega_\pm^2 = k^2 \pm \frac{\xi}{M} k^3$$

for the photon (± accounts for right and left circular polarizations)

$$E^2 \simeq m^2 + p^2 \pm \frac{\eta_\pm}{M} p^3$$

for the fermions

$$\eta_\pm^{af} = -\eta_\mp^f$$

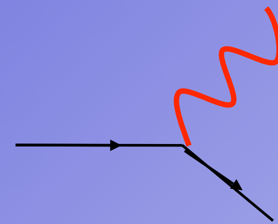
for the antifermions

see **Jacobson et al. Annals Phys. 321 (2006) 150** for more details on the construction of the fermion states…

Threshold for new effects to be active $p_{th} \approx \left(m_e^2\, M/\eta\right)^{1/3} \sim 10\ \mathrm{TeV}\eta^{-1/3}$

Relevant processes

The basic QED vertex is allowed



VC emission: threshold $p_{th} = \left(m_e^2 M / 2\eta\right)^{1/3}$ rate 1 ns

HD: $\left(\eta_+ \neq \eta_-\right)$ $p_{th} = \left(m_e^2 M / \Delta\eta\right)^{1/3}$ effective rate depending on what is the competitive process

Synchrotron:

critical frequency $\omega_c = \frac{3}{2} eB \frac{\gamma^3(E)}{E}$

boost $\gamma(E) \simeq \frac{1}{\sqrt{m^2/E^2 - 2\,\eta E/M}}$

2 regimes

$\eta > 0$ ⟶ γ unbounded, diverges at finite energy (superluminal motion)

$\eta < 0$ ⟶ γ bounded, the critical frequency has a maximum!

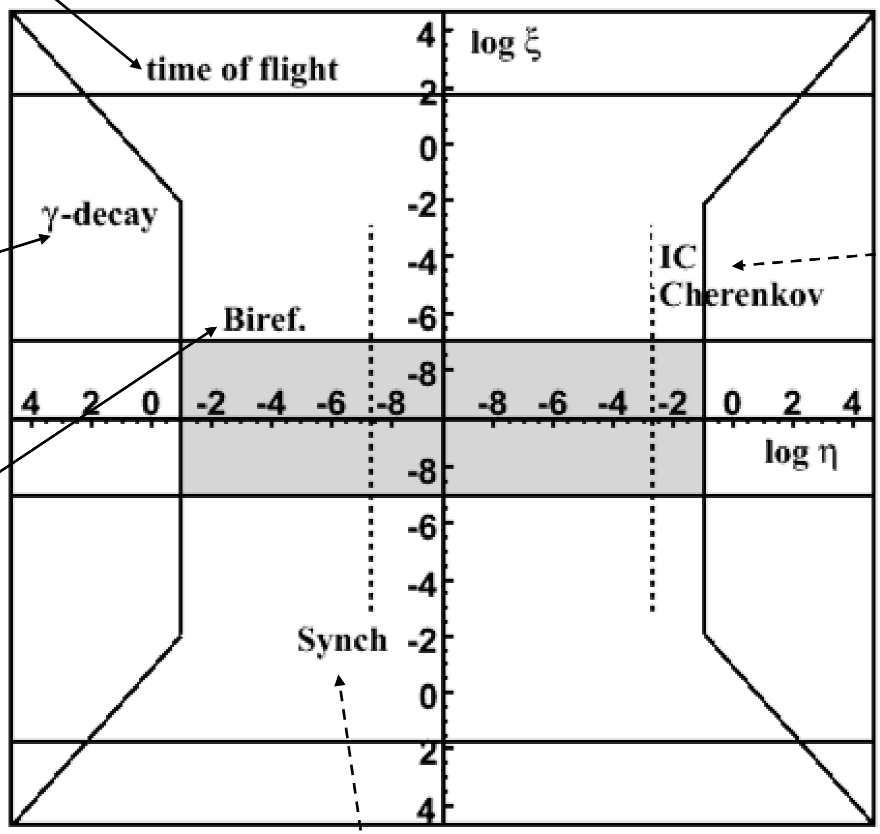
Astrophysical constraints so far…

see **Jacobson et al. Annals Phys. 321 (2006) 150** for more details

delay in arrival time of different colors or polarizations

Observation of 80 TeV photons from the Crab Nebula (HEGRA)

polarization/depolarization of the radiation form a distant source (e.g. UV/Optical from GRB)



Observation of 80 TeV photons from the Crab Nebula (HEGRA)

Not a constraint! Apply just to 1 coefficient each.

Observation of 100 MeV photons from the Crab Nebula (EGRET)

The synchrotron is very sensitive to LV (**Jacobson et al., Montemayor & Urrutia**)

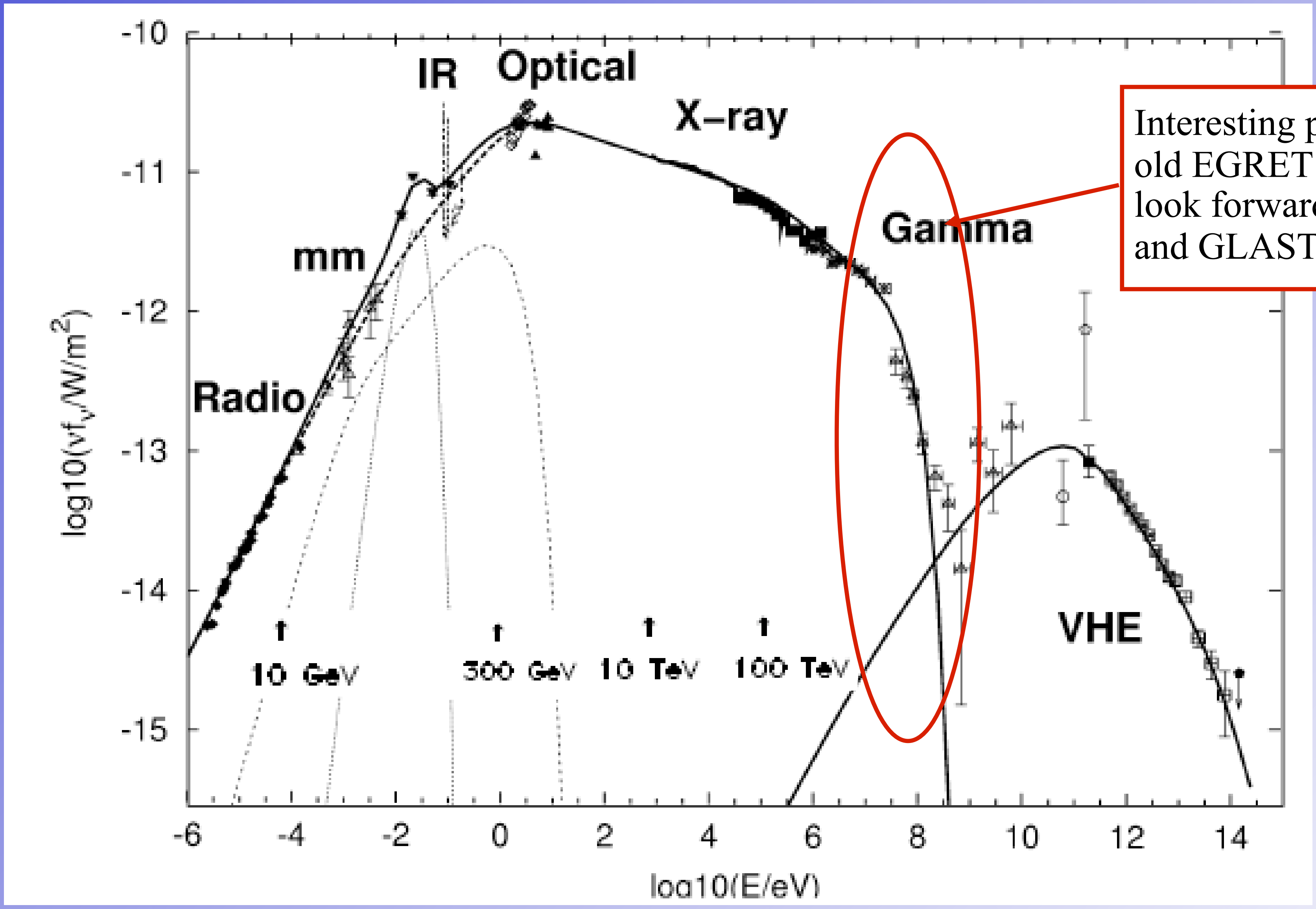
The Crab Nebula

The Crab Nebula – Remnant of a SuperNova explosion

- exploded in **1054 A.D**.
- distance **~1.9 kpc** from Earth
- pulsar wind powered nebula
- most powerful object in the sky
- spectrum spans **21 decades in frequency**, from radio to ~80 TeV
- **leptonic origin** of the radiation
- **electrons accelerated to > PeV**
- **theoretical model** understood only roughly at radio frequencies but enough at >keV energies. (**Kennel & Coroniti, 1984**)

- E < 1 GeV ⟶ **synchrotron**
- E > 1 GeV ⟶ **IC scattering**

The Crab Nebula spectrum



Interesting part:
old EGRET data,
look forward to AGILE
and GLAST results!

Horns et Aharonian, astro-ph/0407119, Aharonian & Atoyan 1998

A new approach

- **Re-compute** the full Crab spectrum relaxing the hypothesis of Lorentz Invariance.
- **Fix most of the free parameters** (magnetic field strength, electron energy density…) from **low frequency observations** (well defined procedure, see later)
- **Check** that LV modifications enter only in the high energy part of the spectrum
- **Compare** with experimental points and make constraints **(chi-square analysis).**

Advantages:

- Can constrain all lepton coefficients
- Exploit the whole information from observations
- More robust constraint (albeit possibly less stringent)

Drawbacks:

- Need a model for the Nebula
- Need to understand how the processes taking place in the Nebula are affected by LV

More details: **L.M., S.Liberati, A.Celotti, J.Kirk, arXiv:0707.2673**

LV model for the Crab Nebula - Emitters

Spectrum: 1st order Fermi mechanism ⟶ power-law with spectral index close to -2. Since it is essentially a kinematic mechanism, we assume the power-law spectrum to be in γ rather than in energy.

$$n(E) \to n(\gamma(E)) = \gamma^{-p} \qquad \mathbf{p \sim 2.2 - 2.4} \qquad \gamma \neq E/m$$

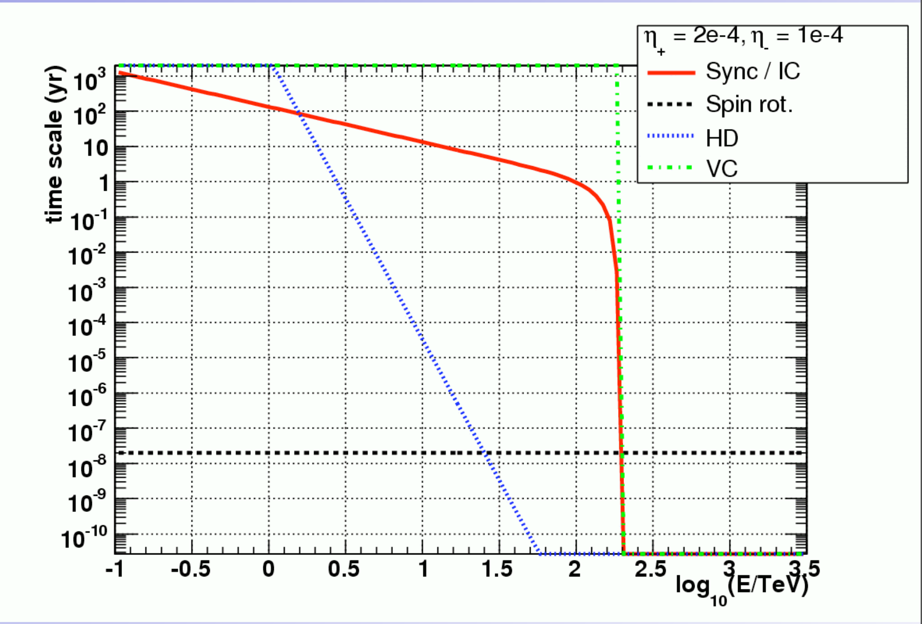
Populations:

charge

	low energy (E < few 100 GeV)	high energy (E > 1 TeV)
e^-_+	1/4	0
e^-_-	1/4	1/2
e^+_-	1/4	0
e^+_+	1/4	1/2

"helicity" state

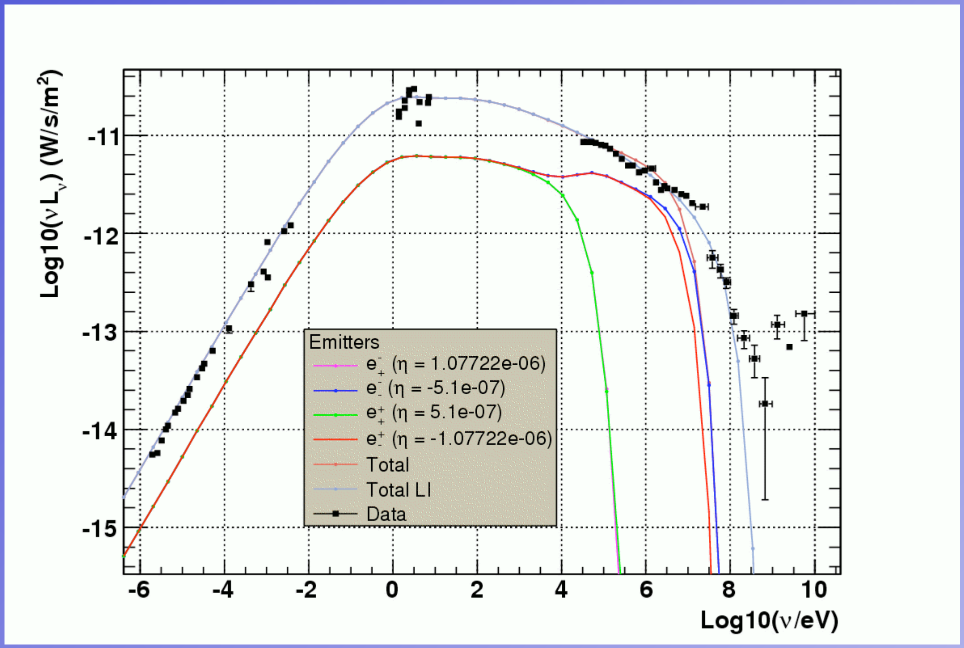
$\eta_+ > \eta_-$



High energy cut-off: set by Vacuum Cherenkov $E_{VC} = \left(m^2 M / 2\eta\right)^{1/3} \approx 10\ \mathrm{TeV}\,\eta^{-1/3}$

LV model for the Crab Nebula - Spectrum

Total spectrum obtained by integrating in a spherical region of radius 1.8 pc

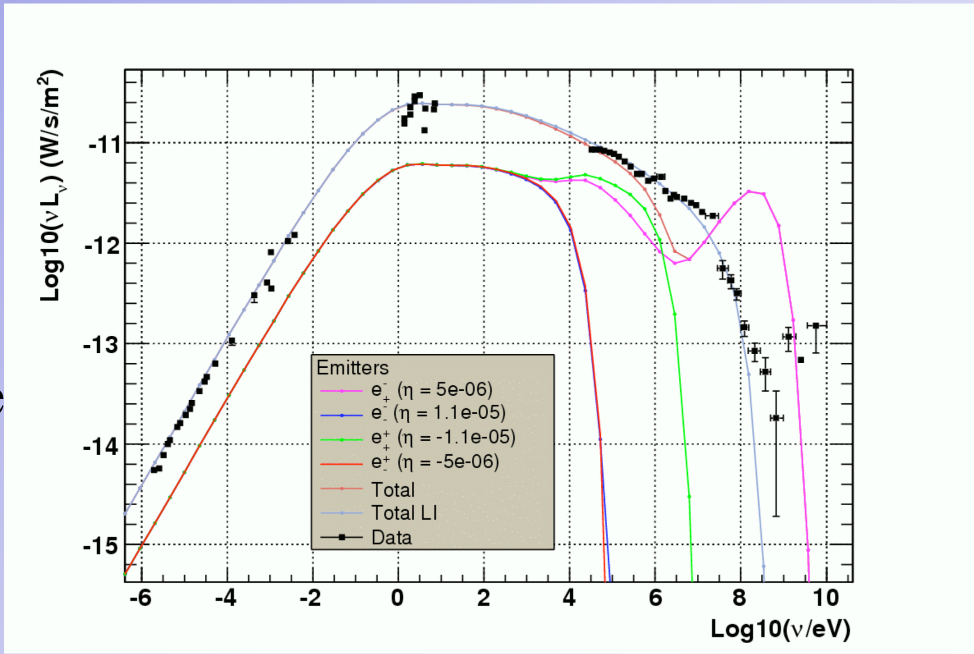


Case A: $\eta_+ > 0, \eta_- < 0$

HD $\longrightarrow$ only particles with negative coefficients survive

Case B: $\eta_+ > 0, \eta_- > 0$

high energy features, due to the divergence of the synchrotron critical frequency.



Constraints

We adopt a χ^2 strategy to impose constraints on LV parameters.

$$\eta_+ \cdot \eta_- > 0$$

90% CL exclusion

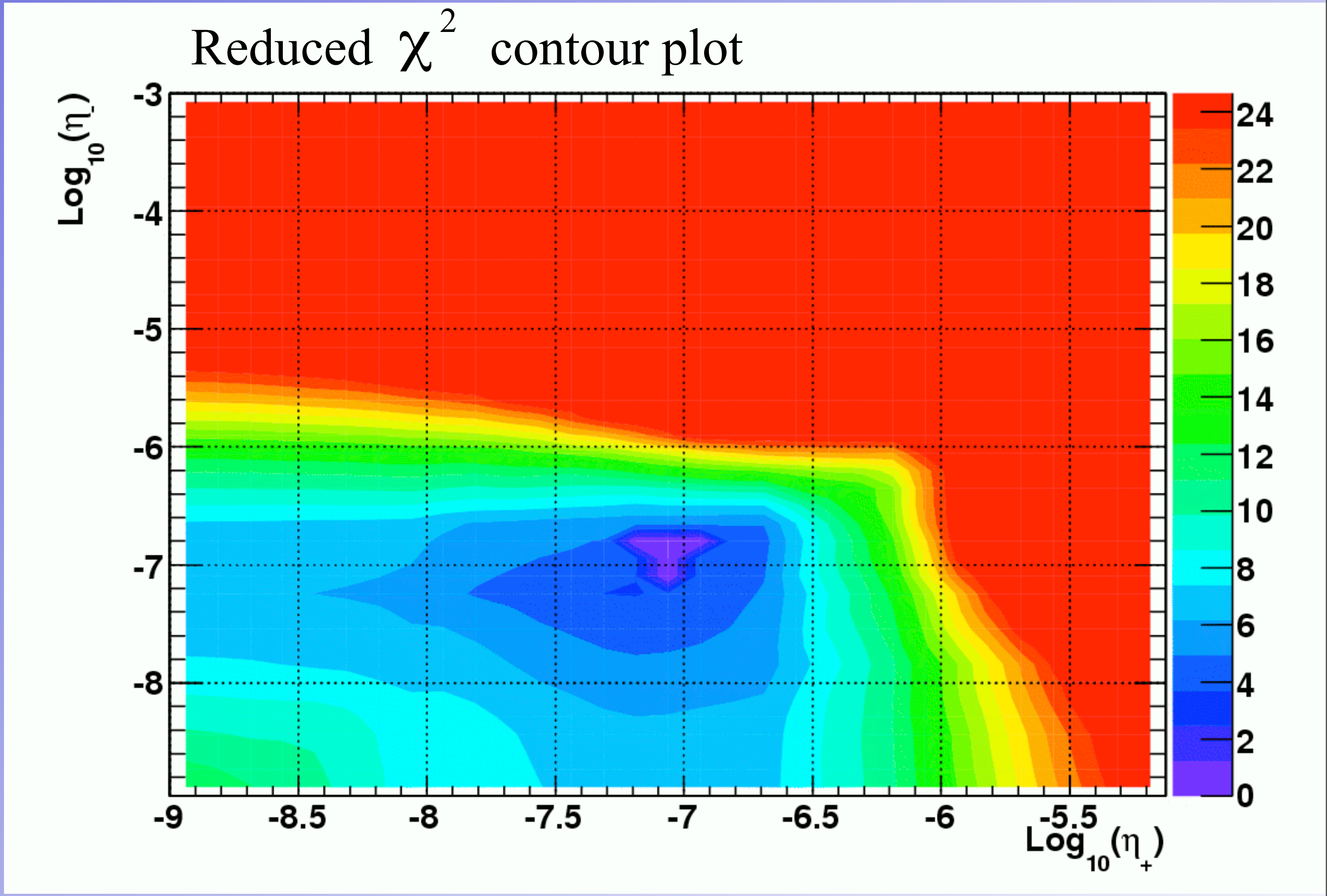
$$\chi^2 > 8$$

95% CL exclusion

$$\chi^2 > 10$$

99% CL exclusion

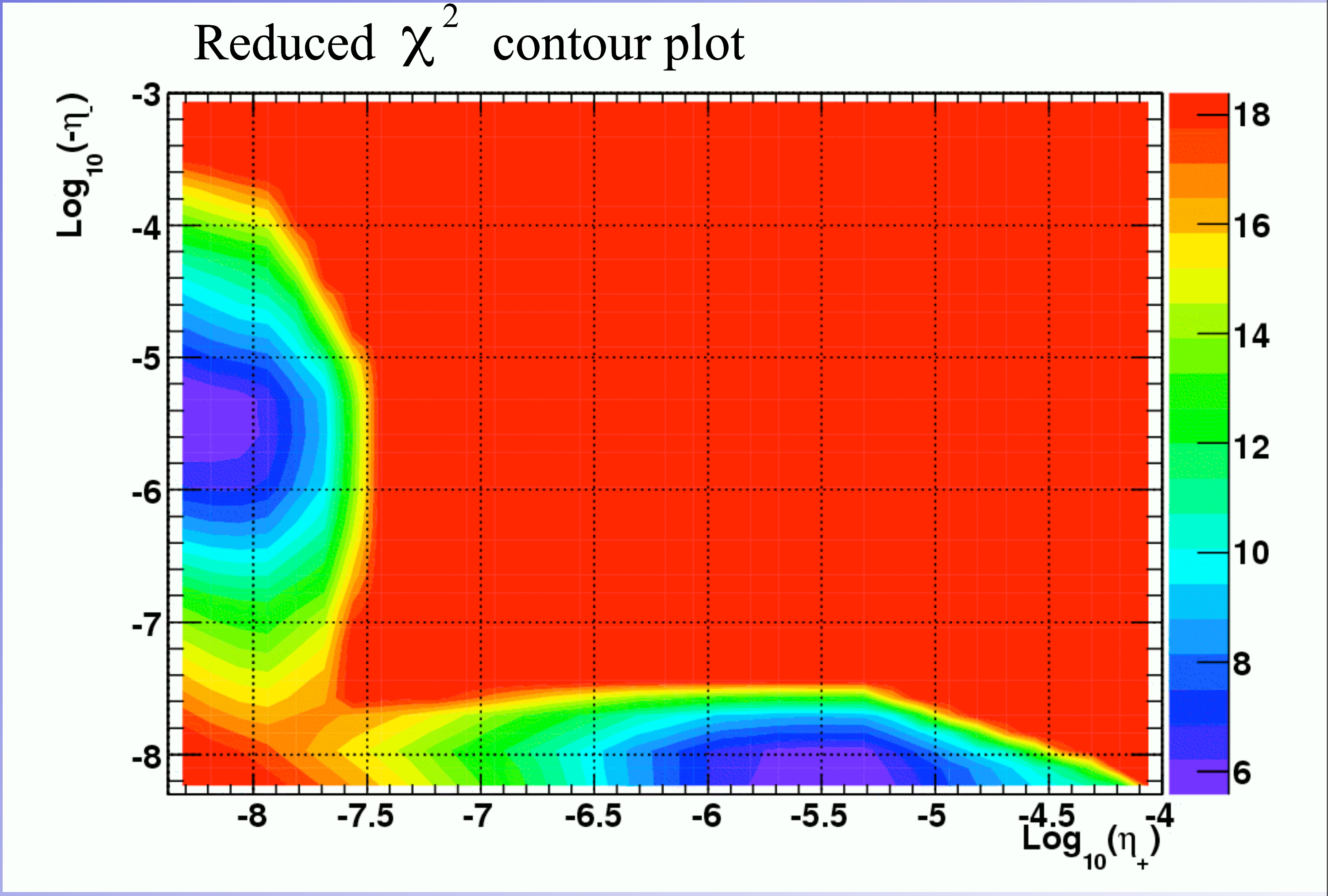
$$\chi^2 > 13$$



O(E/M) LV is excluded at 95% CL at the 10^{-6} level in this case

Constraints

$$\eta_+ \cdot \eta_- < 0$$



90% CL exclusion

$$\chi^2 > 8$$

95% CL exclusion

$$\chi^2 > 10$$

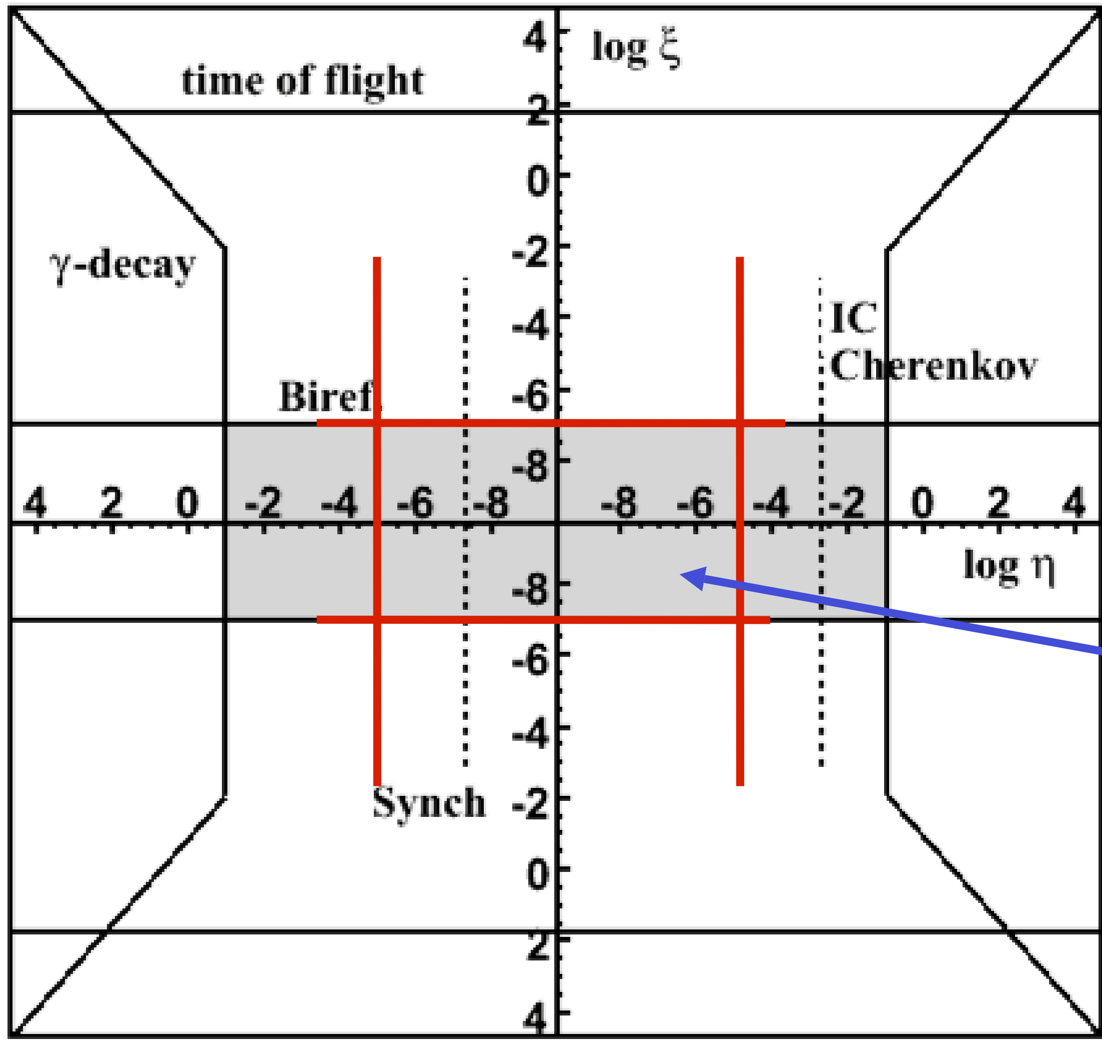
99% CL exclusion

$$\chi^2 > 13$$

O(E/M) LV is excluded at 95% CL at the 10^{-5} level!

New constraints

All coefficients are constrained!!



Allowed region

arXiv:0707.2673

Conclusions

- **It is indeed possible to constrain fundamental theories with high energy astrophysics observations.**

- A **full and self-consistent method** to calculate properties of astrophysical objects in presence of Lorentz Invariance Violation has been understood and coded.

- Many **new effects** have been taken into account.

- **Tight constraints** have been cast to O(E/M) LIV using present data.

- Improvements are expected when GLAST will be flying (have constraints of same order in any case)

- Further **progresses** can be achieved to constrain $\mathbf{O(E^2/M^2)}$ (CPT preserving) LIV

Backup slides

Renormalization Group effects

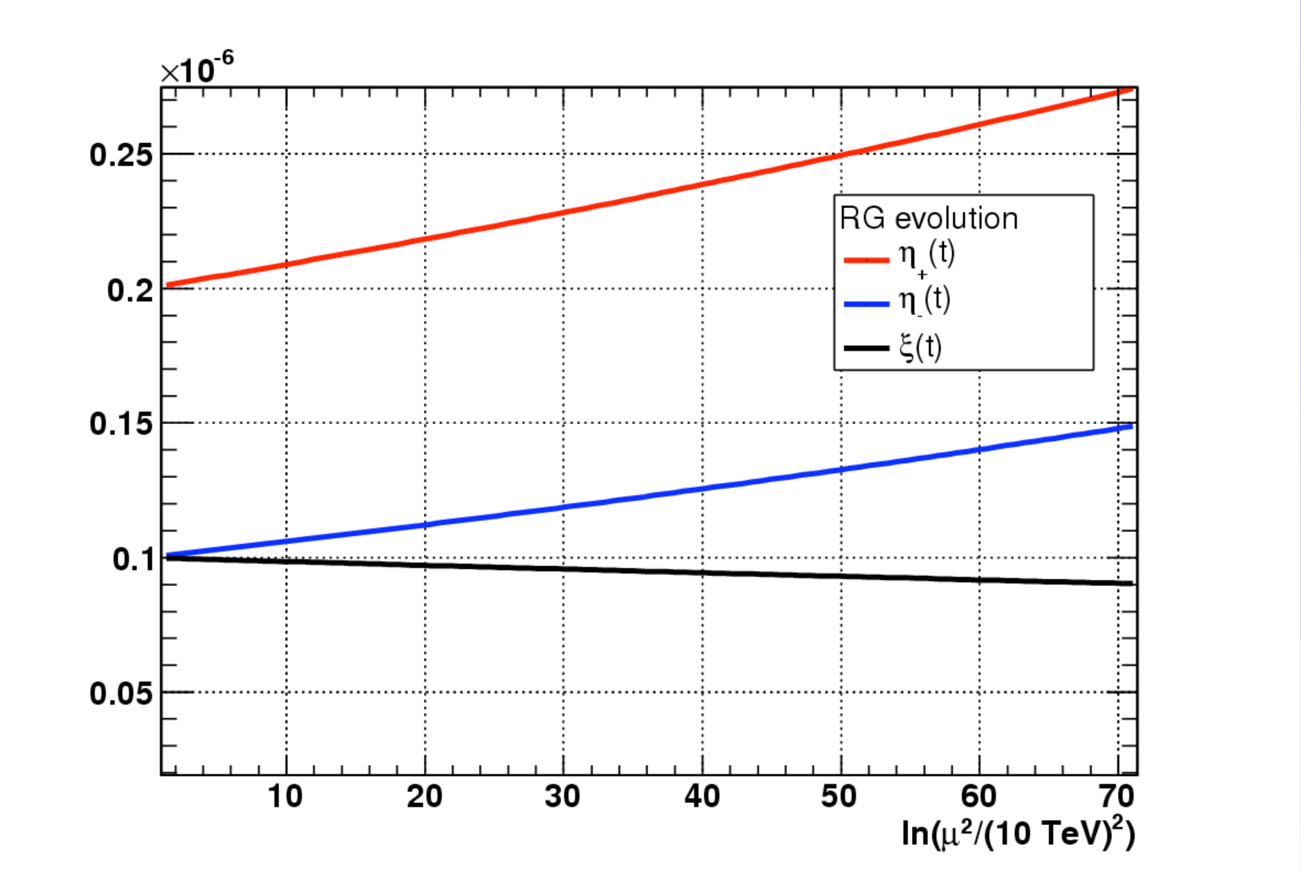
Is the constraint robust against RG effects?

RG equations in M&P QED recently computed (**Bolokhov & Pospelov, hep-ph/0703291**)

$$\begin{cases} \dfrac{d\zeta_1}{dt} = \dfrac{25}{12}\dfrac{\alpha}{\pi}\zeta_1 \\ \dfrac{d\zeta_2}{dt} = \dfrac{25}{12}\dfrac{\alpha}{\pi}\zeta_2 - \dfrac{5}{12}\dfrac{\alpha}{\pi}\xi \\ \dfrac{d\xi}{dt} = \dfrac{1}{12}\dfrac{\alpha}{\pi}\zeta_2 - \dfrac{2}{3}\dfrac{\alpha}{\pi}\xi \end{cases}$$

$$\eta_\pm = 2(\varsigma_1 \pm \varsigma_2)$$

The running is only logarithmic



constraint scale → Planck scale

Renormalization Group effects

Is the constraint robust against RG effects? **Yes!!**

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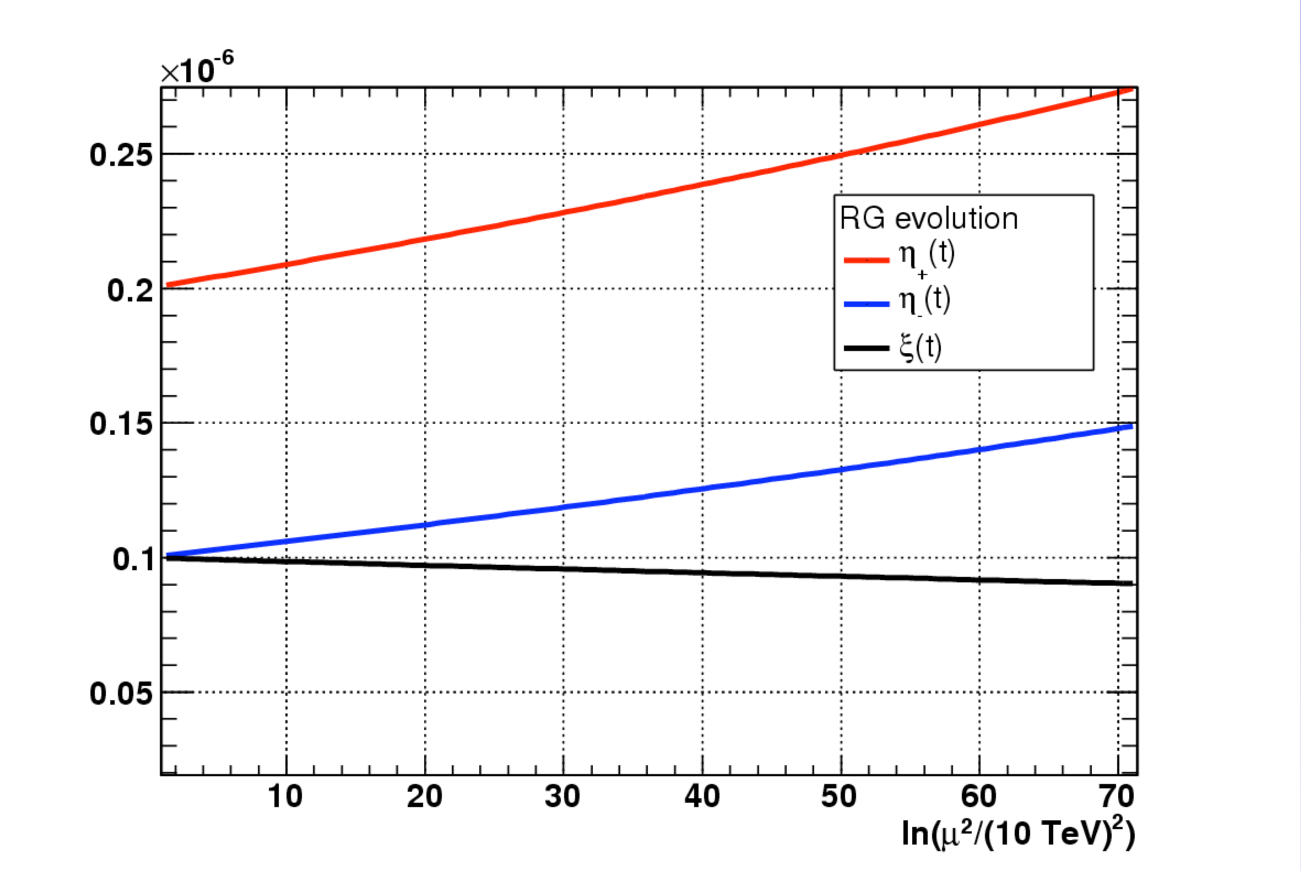

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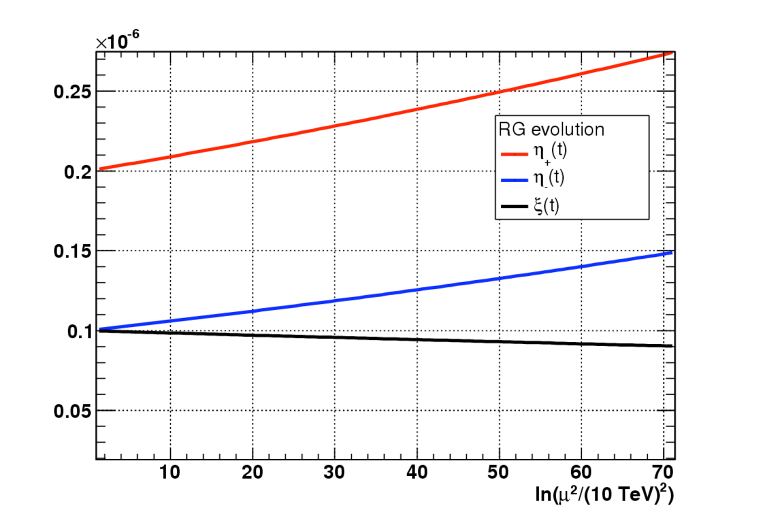
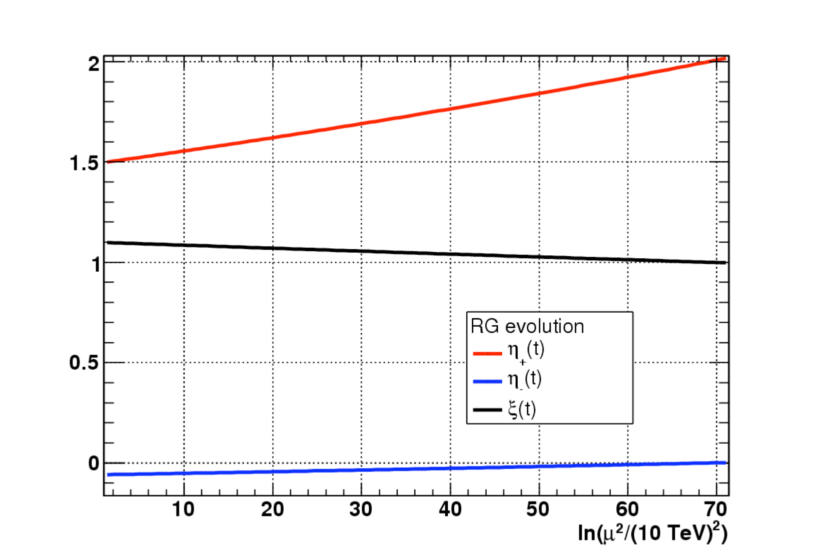
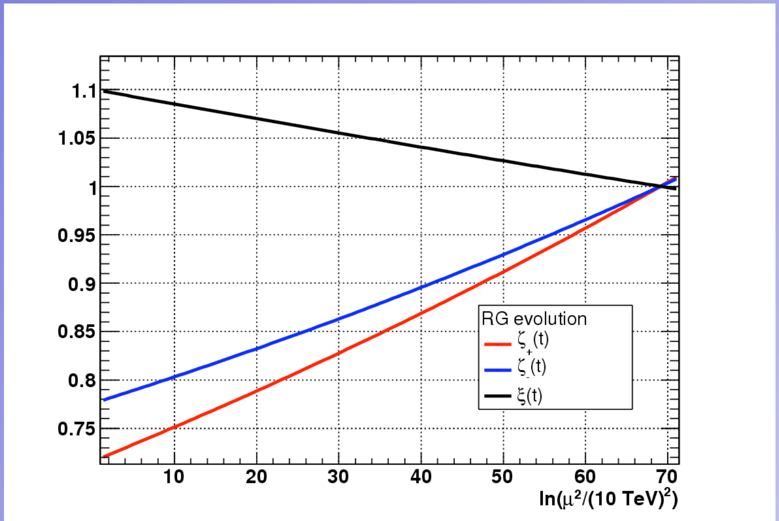
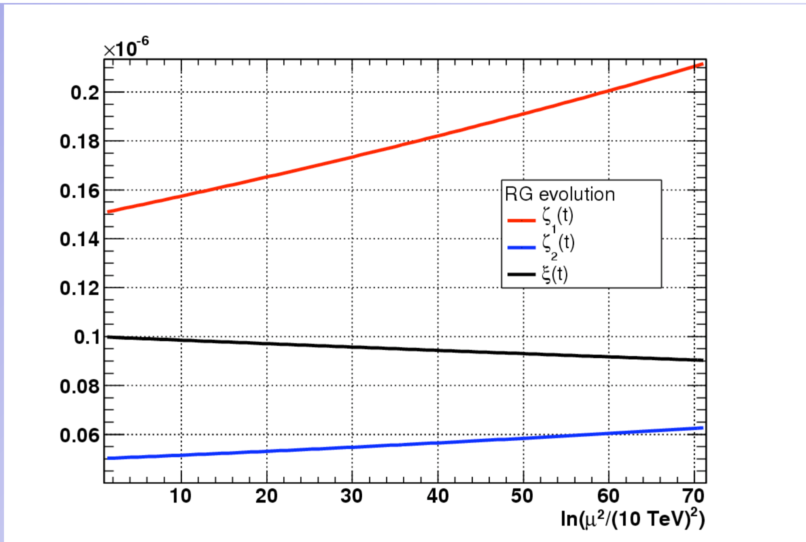
Renormalization Group effects









Model for the Crab Nebula - Emitters

MagnetoHydroDynamic model developed by KC

2 populations of electrons responsible of the spectrum (just a working assumption)
- **radio (relic) electrons**: responsible for the radio to optical radiation
  they have been cooled down by synchrotron emission ($t \approx 10^3\ \mathrm{yr}(E/50\ \mathrm{GeV})^{-1}$)
  Tipical energy < **200 GeV**

$$n_{re}(E) = A_{re} E^{-p_r} e^{-E/E_*}$$

- **wind electrons**: freshly accelerated (by Fermi 1st order mechanism) electrons injected into the Nebula at a wind termination shock and then propagated across the Nebula. Responsible for the X- to VHE- radiation.

$$n_s(E) = A_{we}(E_0 + E)^{-p_w} e^{-E/E_c}$$

Beyond Lorentz Invariance

Most popular frameworks

Lorentz Symmetry Breaking

A preferred frame exists!

It is the rest frame of CMB

We live where deviations from Lorentz Symmetry are small.

Ex:
LIV in Effective Field Theory
(e.g. **Colladay & Kostelecký, 1998**
Myers & Pospelov, 2003)

suitable for constraints: well defined low energy theory!

Alternatively:

Doubly Special Relativity.

The Lorentz Group acts in a non-linear way on the fields.

2 invariant quantities

c ⟶ the speed of light

λ^{DSR} ⟶ an energy scale

"deformed" Lorentz Symmetry

only defined in momentum space

lack of a formulation in configuration space

not suitable for constraints: what is the low energy dynamics?

Beyond Lorentz Invariance

Our framework

Dimension 5 Standard Model Extension: include dimension 5 LV operators in the SM preserving gauge and rotation invariance and quadratic in the fields
Myers & Pospelov, 2003
Contribution at order **p^3/M** to the MDR
(neglect lower order terms, that have strong experimental bounds).

$$L = L_{QED} - \frac{\xi}{2M} u^m F_{ma} (u \cdot \partial)(u_n \tilde{F}^{na}) + \frac{1}{2M} u^m \bar{\psi} \gamma_m (\varsigma_1 + \varsigma_2 \gamma_5)(u \cdot \partial)^2 \psi$$

u^m = timelike constant vector.

Note! All these terms also violate CPT

Modified Dispersion Relations

$$\omega_\pm^2 = k^2 \pm \frac{\xi}{M} k^3$$

for the photon (± accounts for right and left circular polarizations)

$$E^2 \simeq m^2 + p^2 \pm \frac{\eta_\pm}{M} p^3$$

for the fermions $\eta_\pm = 2(\varsigma_1 \pm \varsigma_2)$

$$\eta_\pm^{af} = -\eta_\mp^{f}$$

for the antifermions

see **Jacobson et al. Annals Phys. 321 (2006) 150** for more details on the construction of the fermion states…

Outlook

Outlook:

- **motivations** for relaxing Lorentz Invariance assumption
- **phenomenology** of LIV in Effective Field Theory
- **present constraints** and the **Crab Nebula**
- **LV modelisation** in astrophysical context
- **results and conclusions**