The dark matter distribution on small scales

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- Why?
- Early Universe microphysics
- The first WIMPy halos
- Their evolution & present day distribution
- Implications/open questions
- What about non-WIMPy DM candidates?

Why?

Simulations produce halos containing large amounts of substructure:
[Klypin et al.; Moore et al.;........ Diemand, Kuhlen & Madau]

$$\frac{\mathrm{d}n}{\mathrm{d}m} \propto m^{-\alpha} \qquad \alpha \sim 2$$

down to the resolution limit ($\sim 10^6 M_\odot$ for a Milky Way-like halo), with ~5-10% of the total mass in (resolved) sub-structure.

What happens on smaller scales?

Does this mass function carry on down to infinitesimally small scales?/How big are the first DM halos to form?
(n.b. there must be a cut-off at some point, otherwise the contribution of the density perturbations to the local energy density would diverge)

What fraction of the total mass is in substructure?

Also interesting/important for practical reasons:

- ## Indirect detection

Event rates proportional to ρ^2, enhanced by sub-structure.
[Silk & Stebbins; Bergström et al.; Calcaneo-Roldan & Moore;Ullio et al.; Taylor & Silk....]

Nearby mini-halos easier to detect than nearest larger subhalo (smaller distance outweighs smaller mass)?

- ## Direct detection

Signals depends on the dark matter distribution on sub-milli-pc scales. [Silk & Stebbins; Moore et al.; AMG]

n.b. The 'Halo models' which are often used in direct detection calculations are solutions of the collisionless Boltzmann equation- this applies to the coarse grained (i.e. spatially averaged) distribution function and assumes the dark matter distribution has reached a steady state.

WIMP microphysics

[Schmid, Schwarz & Widerin; Boehm, Fayet & Schaeffer; Chen, Kamionkowski & Zhang; **Hofmann, Schwarz & Stöcker**; Schwarz, Hofmann & Stöcker; Berezinsky, Dokuchaev & Eroshenko; AMG, Hofmann & Schwarz x2; Loeb & Zaldarriaga; Bertschinger; Bringmann & Hofmann]

Kinetic decoupling

After freeze-out (chemical decoupling) WIMPS carry on interacting kinetically with radiation:

$$\chi+\chi \not\Leftrightarrow X+X \qquad\qquad \chi+X \Leftrightarrow \chi+X$$

The WIMPs kinetically decouple when

$$\tau_{\rm relax} = H^{-1}$$

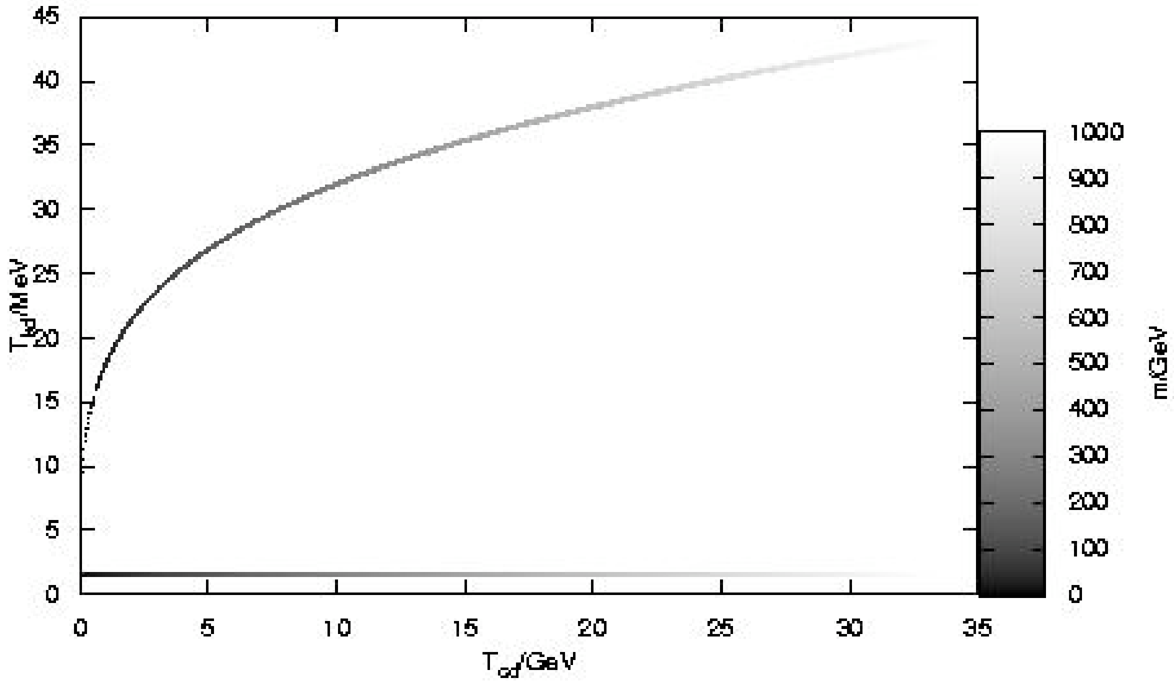
n.b. the momentum transfer per scattering (~T) is small compared with the WIMP momentum (~M), therefore a very large number of collisions are required to keep or establish thermal equilibrium.

$$\tau_{\rm relax} \gg \tau_{\rm col}$$

Dependence of decoupling temperatures on WIMP mass, for WIMPs with present day density compatible with WMAP measurements, for l =1 for Majorana particles (i.e. neutralinos interacting via sfermion exchange) and l=0 Dirac particles (i.e. standard modelesque particles interacting via Z0 exchange).

$$< \sigma_{\rm el} >= \sigma_0^{\rm el} \left(\frac{T}{m}\right)^{l+1} \qquad \sigma_0^{\rm el} \approx \frac{(G_{\rm F} m_{\rm W}^2)^2 m^2}{m_{\rm Z}^2}$$

Kinetic decoupling temperature in MeV



mass in GeV

Chemical decoupling temperature in GeV

Collisional damping

Energy transfer between radiation and WIMP fluids (due to bulk and shear viscosity) leads to collisional damping of density perturbations.

Free-streaming

After kinetic decoupling WIMPs free-stream, leading to further (collision-less) damping.

$$\frac{1}{k_{\rm fs}} \sim l_{\rm fs}(\eta) = \bar{v}_{\rm kd} a_{\rm kd} \int_{\eta_{\rm kd}}^{\eta} \frac{{\rm d}\eta'}{a(\eta')}$$

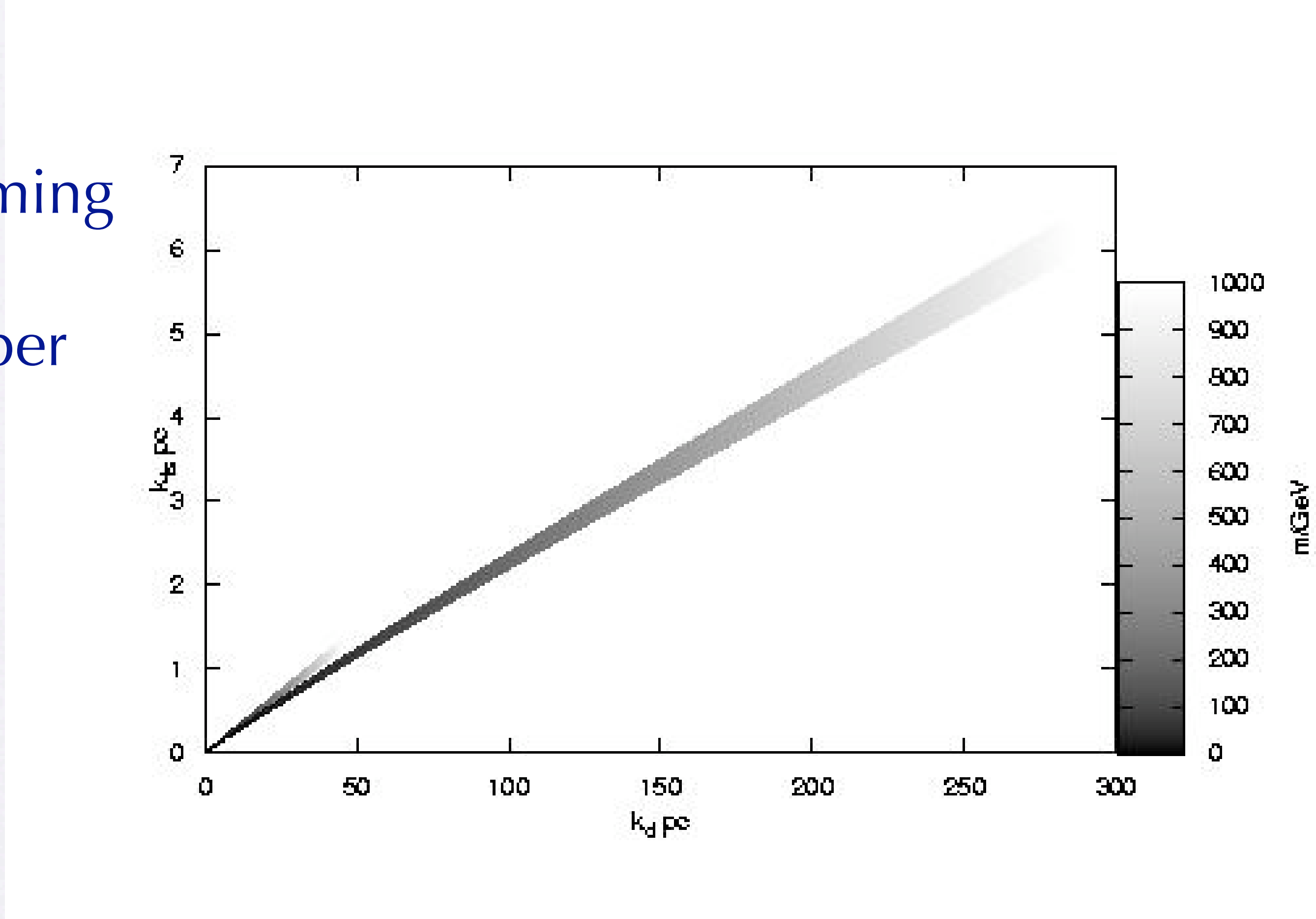
Calculate free-streaming length by solving the collisionless Boltzmann equation, taking into account perturbations present at kinetic decoupling.

Net damping factor:

$$D(k) \equiv \frac{\delta_{\rm WIMP}(k,\eta)}{\delta_{\rm WIMP}(k,\eta_{\rm i})} = D_{\rm cd}(k) D_{\rm fs}(k) = \left[1 - \frac{2}{3}\left(\frac{k}{k_{\rm fs}}\right)^2\right] \exp\left[-\left(\frac{k}{k_{\rm fs}}\right)^2 - \left(\frac{k}{k_{\rm d}}\right)^2\right]$$

Dependence of damping scales on WIMP mass, for WIMPs with present day density compatible with WMAP measurements, and *l* =0/1 (top and bottom).

Free-streaming comoving wavenumber (pc)



mass in GeV

Collisional damping comoving wavenumber (pc)

WIMP micro-physics summary

- ✩ $T > T_{cd}$ [O(1-10) GeV]

  In chemical and thermal equilibrium

- ✩ $T = T_{cd}$

  Chemical decoupling/freeze-out, comoving number density becomes fixed.

- ✩ $T_{kd} < T < T_{cd}$

  Interact kinetically with radiation. Perturbations collisionally damped due to bulk and shear viscosity.

- ✩ $T = T_{kd}$ [O(1-10) MeV]

  Kinetic decoupling, free-streaming regime commences.

- ✩ $T_{eq} < T < T_{kd}$

  Free-streaming erases further perturbations.

Two more ingredients needed to calculate the (processed) density perturbation power spectrum:

- Primordial power spectrum

  Simplest possibility: scale invariant (n=1), WMAP normalised.

- Gravitational growth of fluctuations

Solved perturbation equations for $k \gg k_{\rm eq} \sim 0.01/{\rm Mpc}$ for 2 overlapping regimes:

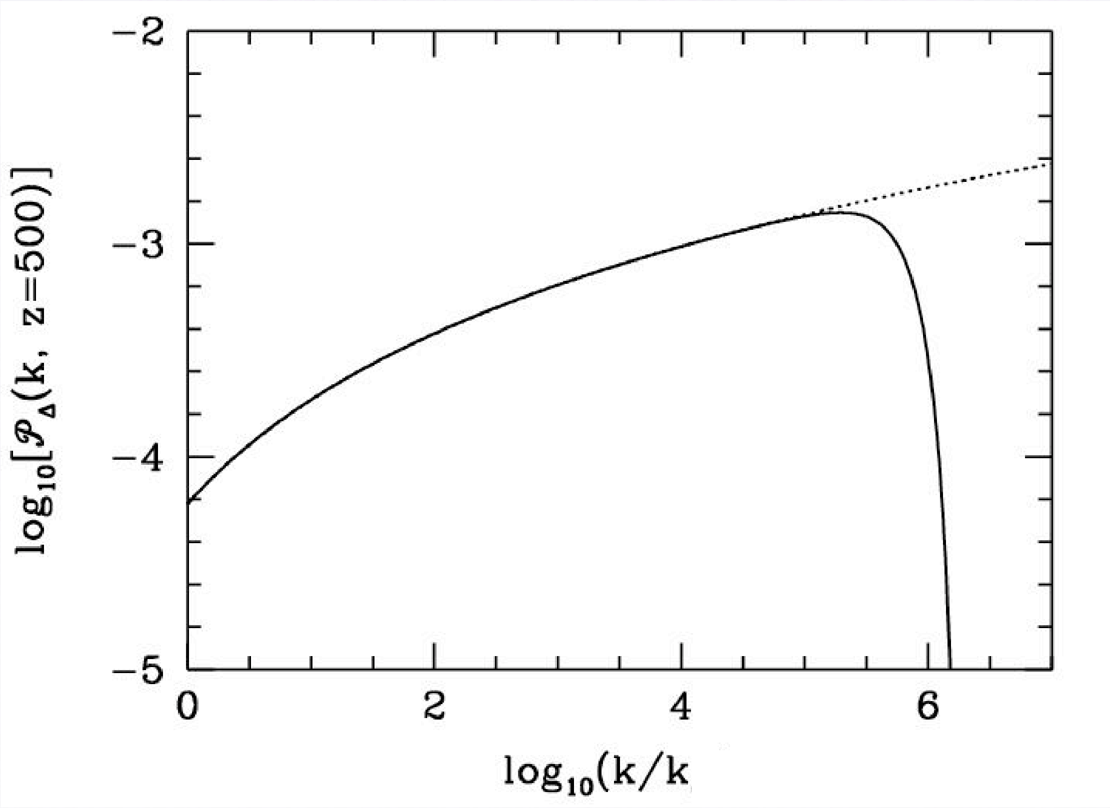
i) radiation domination $\rho_{\rm rad} \gg \rho_{\rm mat}$

ii) $\rho_{\rm mat}\delta_{\rm mat} \gg \rho_{\rm rad}\delta_{\rm rad}$ (Meszaros equation)

(included growth supression due to baryons and verified accuracy of solutions using COSMICs package [Bertschinger])

Power spectrum

For a 100 GeV bino-like WIMP and a scale invariant, WMAP normalised, primordial power spectrum at z=500:



$$\mathcal{P}_\delta(k) = \frac{k^3}{2\pi^2}\langle|\delta^2|\rangle$$

Sharp cut-off at $k = k_{fs} \sim 1/pc$

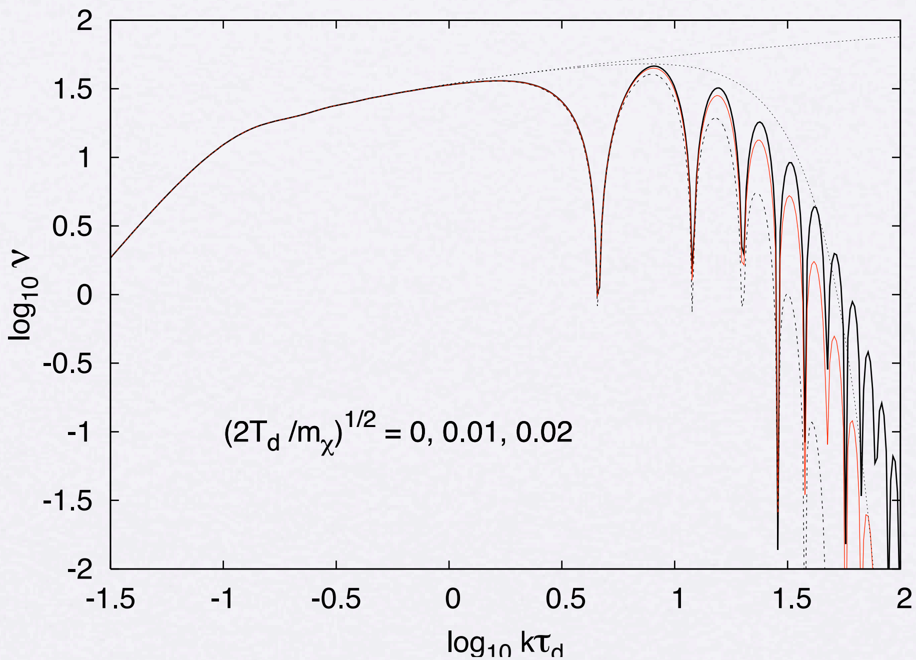
Refinements:

Loeb & Zaldarriaga:

Memory of coupling to radiation fluid leads to accoustic oscillations of CDM fluid and additional damping.

Bertschinger:

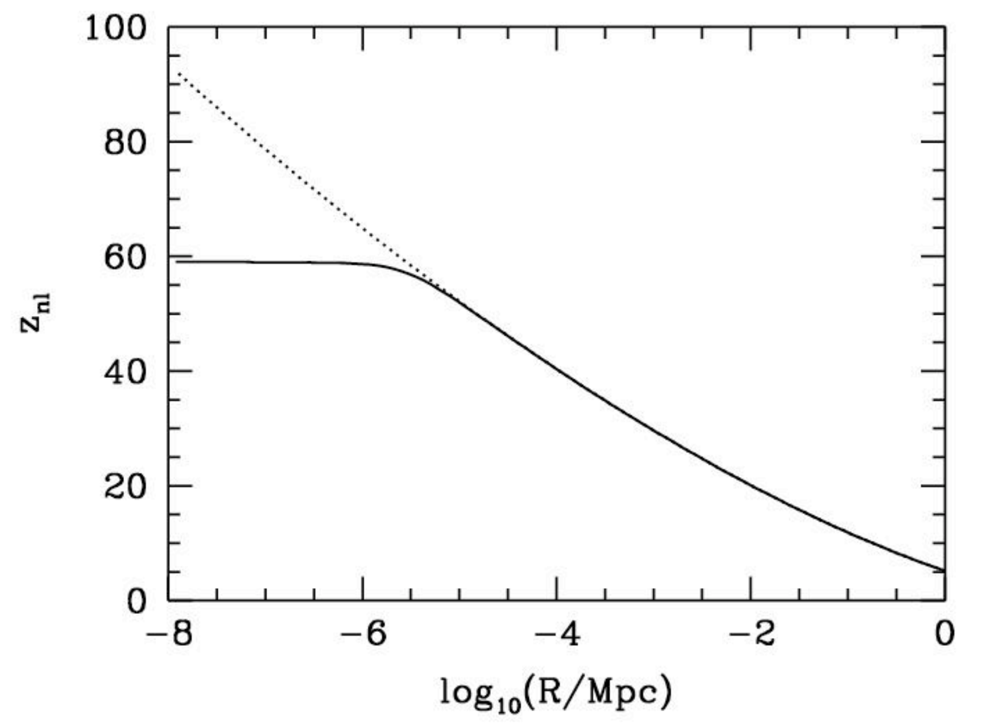
Numerical solution of Fokker-Planck equation for WIMP-lepton interactions.



<10% accurate calculation of the cut-off scale and the detailed shape of the processed power spectrum requires numerical calculations.

z_{nl}

The red-shift at which typical fluctuations on co-moving physical scale R go non-linear can be estimated via the mass variance:



$$\sigma(R, z_{\rm nl}) = 1$$

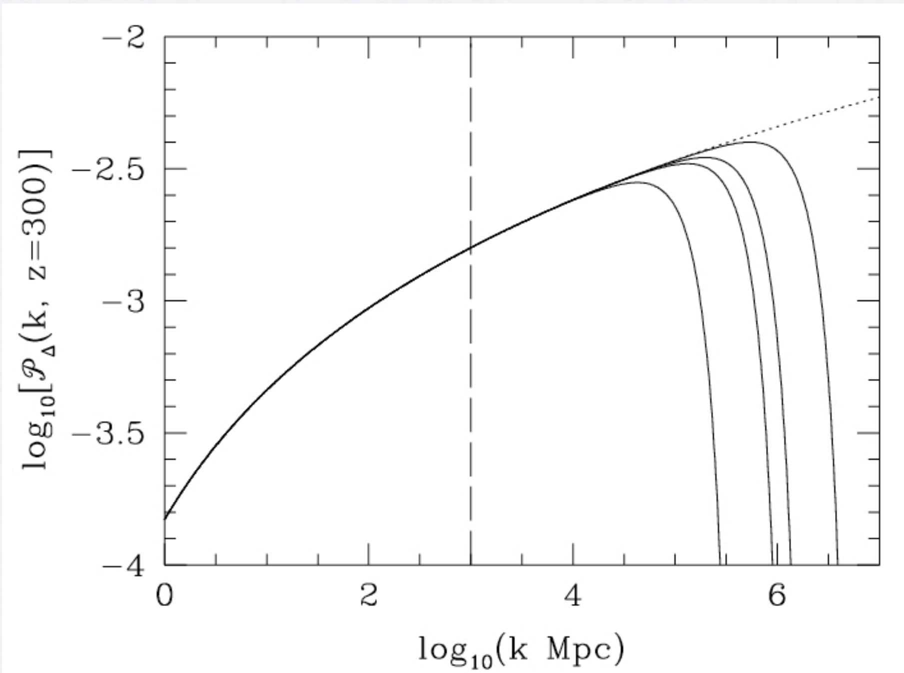
$$\sigma^2(R, z) = \int_0^\infty W^2(kR)\mathcal{P}_\delta(k, z)\frac{\mathrm{d}k}{k}$$

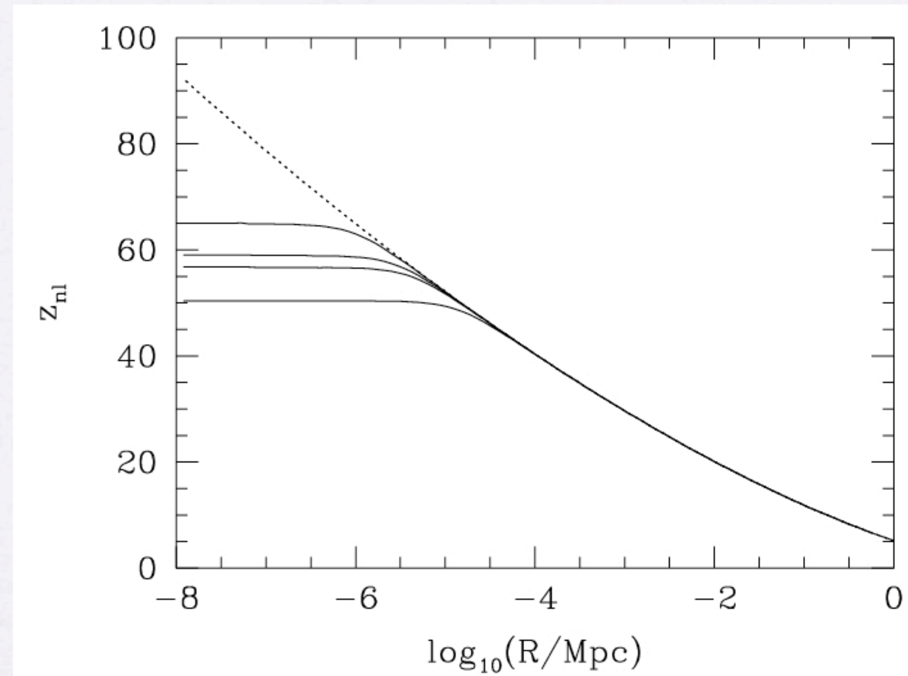
Typical one-sigma fluctuations collapse at z_{nl}~60.

(N-sigma fluctuations collapse at z_{nl}~ 60N)

Effect of varying:

i) WIMP properties





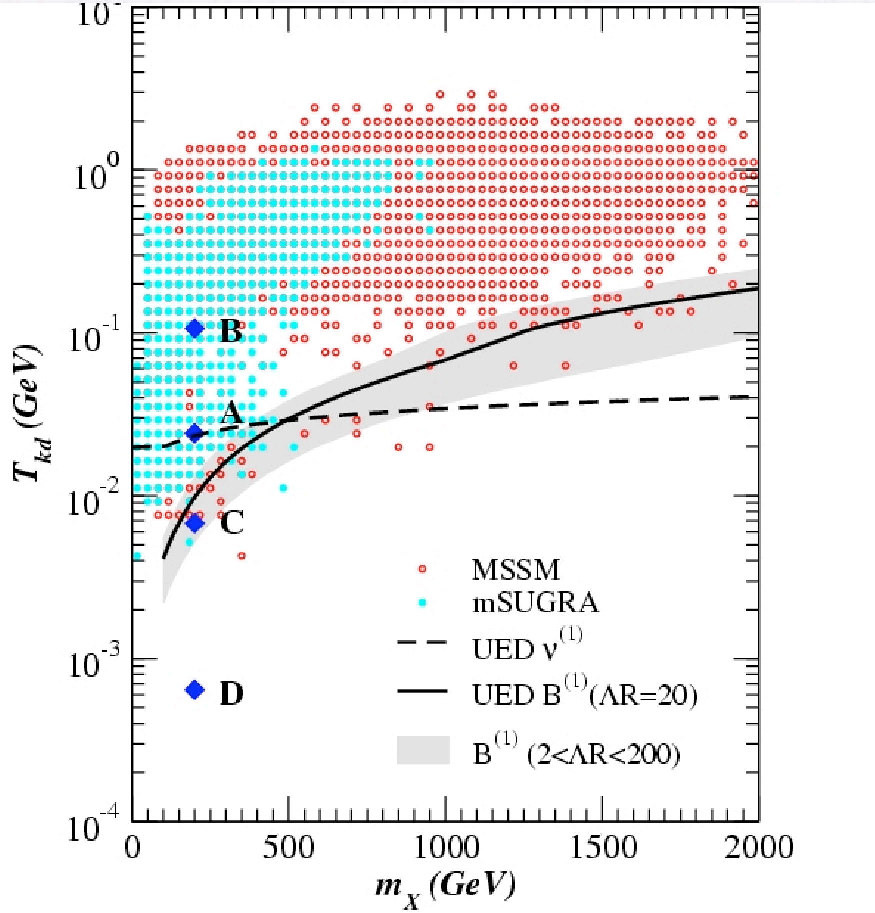
left to right/bottom to top:

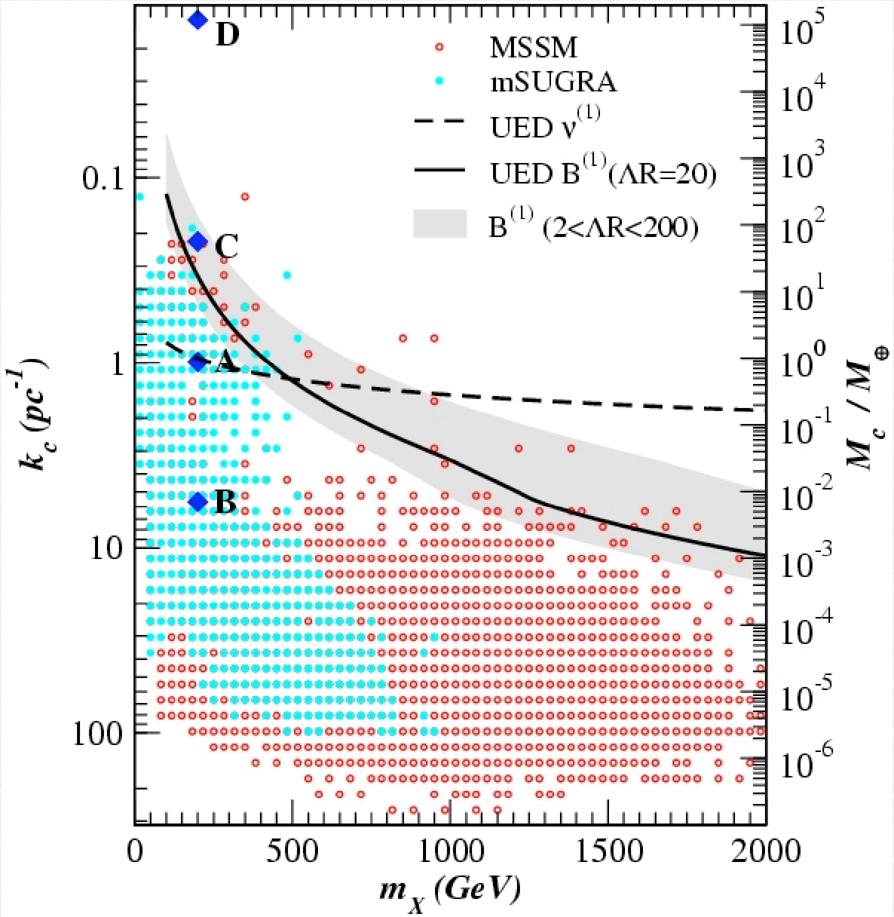
Dirac (elastic scattering mediated by Z_0 exchange) m = 100 GeV

Majorana (Z_0 exchange supressed) m = 50, 100, 500 GeV

Profumo, Sigurdson & Kamionkowski:

Scan MSSM and also consider Universal Extra Dimensions [see also Bringmann & Hofmann] and heavy neutrino like dark matter.





A: coannihilation region, light scalar sparticles, (quasi-degenerate) NLSP is stau

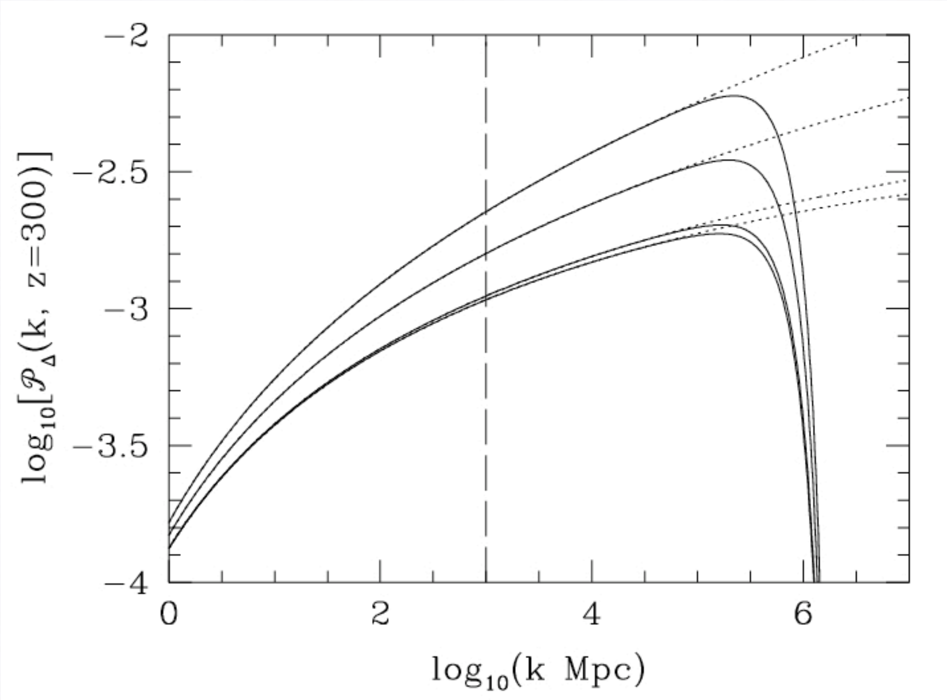
B: focus point region, heavy scalars, scattering from light fermions is via Z0 exchange

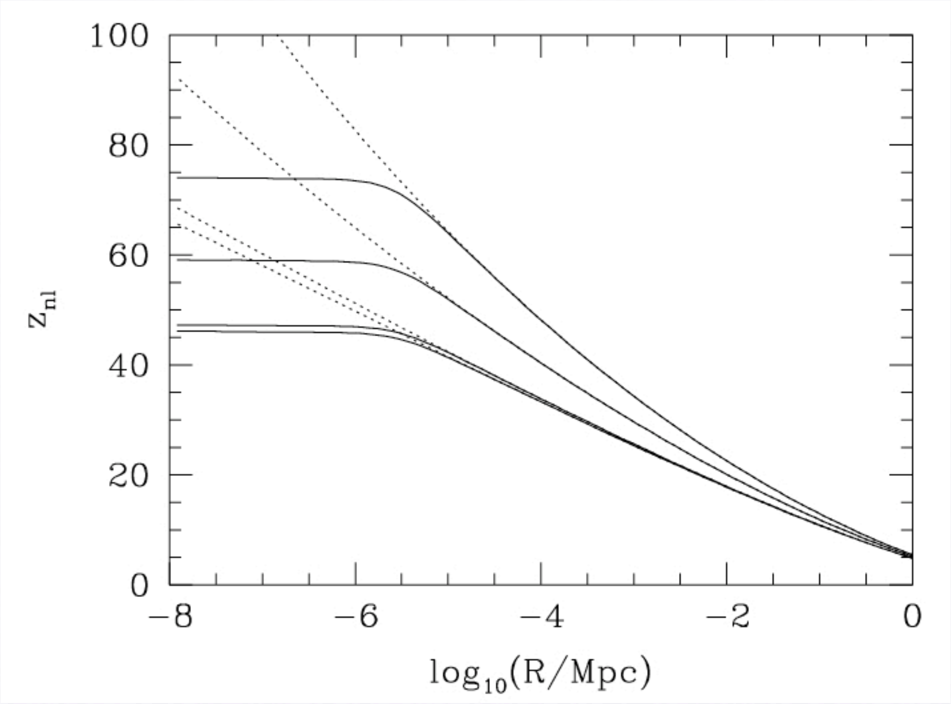
C: $\Delta m_{\tilde{\nu}_{e,\mu}} \equiv m_{\tilde{\nu}_{e,\mu}} - m_\chi = 1\,\mathrm{GeV}$

D: $\Delta m_{\tilde{\nu}_{e,\mu}} \equiv m_{\tilde{\nu}_{e,\mu}} - m_\chi = 0.01\,\mathrm{GeV}$

} Sfermion resonances. At high T scattering from light fermions energy independent.

ii) primordial power spectrum





top to bottom:

false vacuum dominated hybrid inflation	n=1.036, α=0
scale invariant	n=1.000, α=0
power law inflation	n=0.964, α=0
$m^2\Phi^2$ chaotic inflation	n=0.964, α=-0.0006

$$\alpha = \frac{dn}{d\ln k}$$

The first WIMPy halos

Spherical collapse model

Estimates of properties: $M \sim 10^{-6} M_\odot$

$$r \sim \frac{0.02}{N} \text{pc}$$

present day density contrast: $\Delta \sim 10^6 N^3$

Simulations

Current state of the art: particle mass ~ $10^5 M_\odot$ for a Milky Way mass halo.

Re-simulation technique:

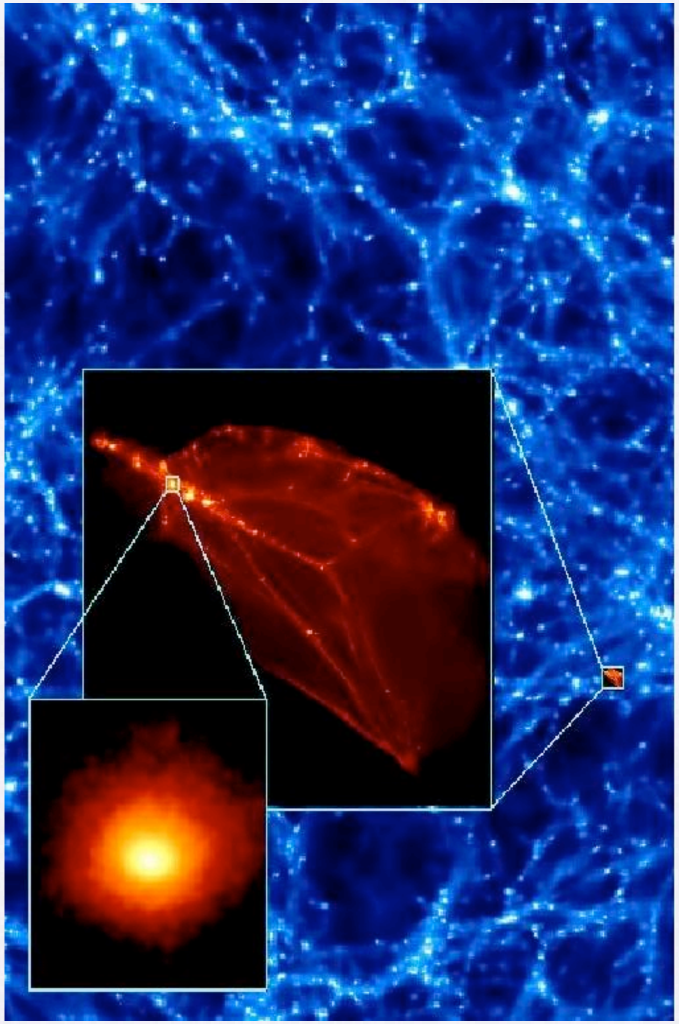
- Extract a region of interest from a cosmological simulation.
- Trace particles back to initial time.
- Re-simulate at higher resolution (smaller particle mass) with surrounding high mass particles to reproduce the tidal forces from the surrounding region.

[Diemand, Moore and Stadel]

Re-simulate a small 'typical' region starting at z=350 (when the fluctuations are still linear) up until z=26 (when the high resolution region begins to merge with surrounding low resolution regions).

Input: Power spectrum with cut-off at k=0.6pc.

Cosmological parameters as measured by WMAP.

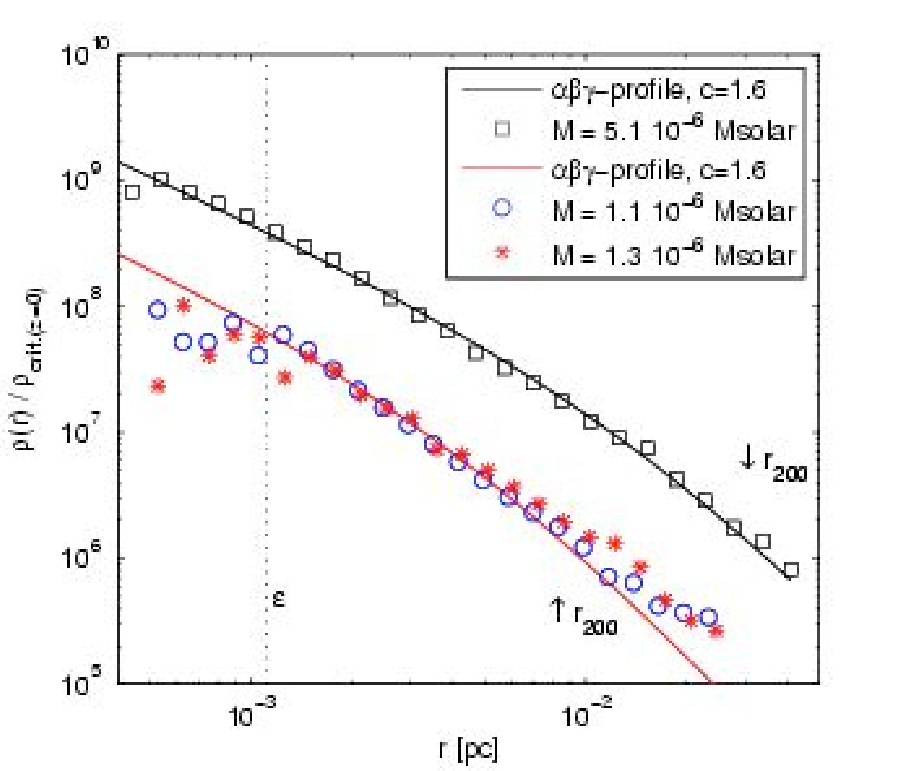


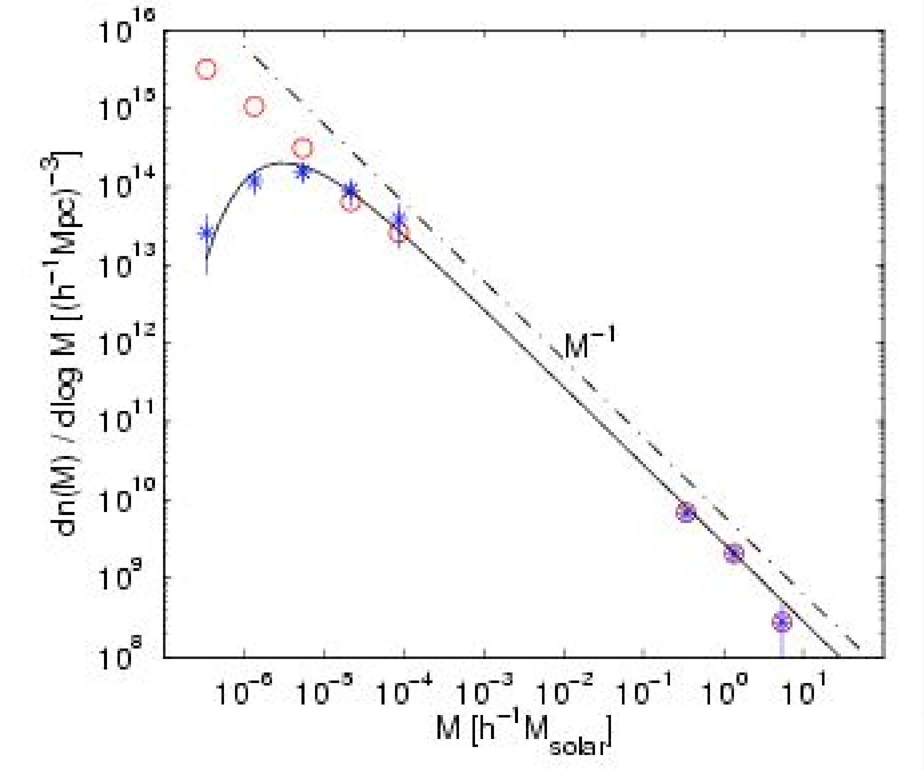
Initial box size $(3\ \mathrm{kpc})^3$
both zooms are x100.

First non-linear structures form at z~60 and have $M \sim 10^{-6} M_{\odot}$

Properties of halos at z=26:

$$\frac{\mathrm{d}n}{\mathrm{dlog}M} \propto M^{-1}$$





Evolution

Various dynamical processes:

Initial hierarchical structure formation

[Berezinsky, Dokuchaev & Eroshenko; Diemand, Kuhlen & Madau]

(processed) power spectrum is weak function of scale:

Similar mass mergers far more common than on Galactic scales.

Difference between density contrast of sub-halos and (immediate) parents smaller

$\Rightarrow$ Most micro-halos destroyed

(but number density of surviving halos is still potentially significant)

Tidal striping

Matter stripped (mainly) from outer-regions if gravitational field of parent halo exceeds field of micro-halo.

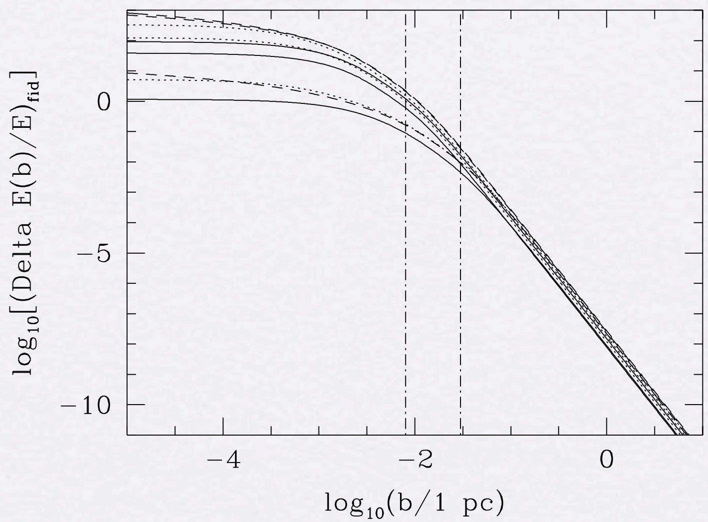
Various authors significant mass loss only within inner few kpc of MW.

Encounters with stars

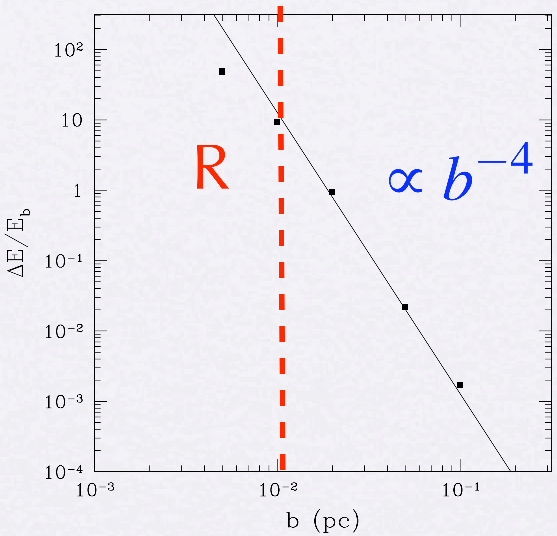
[Zhao, Taylor, Silk & Hooper; Moore, Diemand, Stadel & Quinn; Zhao, Hooper, Angus, Taylor & Silk; Berezinsky, Dokuchaev & Eroshenko; AMG & Goodwin; Goerdt, Gnedin, Moore, Diemand & Stadel; Angus & Zhao]

Micro-halos which pass through the MW disc will be heated (and lose energy/mass) due to encounters with stars.

Duration of encounter much less than micro-halo dynamical time-scale so *energy input* can be accurately calculated analytically using the impulse approximation [for impact parameter b >> or << radius R].



Green & Goodwin.



Goerdt et al.

fractional energy input:

independent of micro-halo mass or density profile for b>> R.

greater for lighter micro-halos (& depends on central density profile) for b <~R.

Goerdt, Gnedin, Moore, Diemand & Stadel; Angus & Zhao

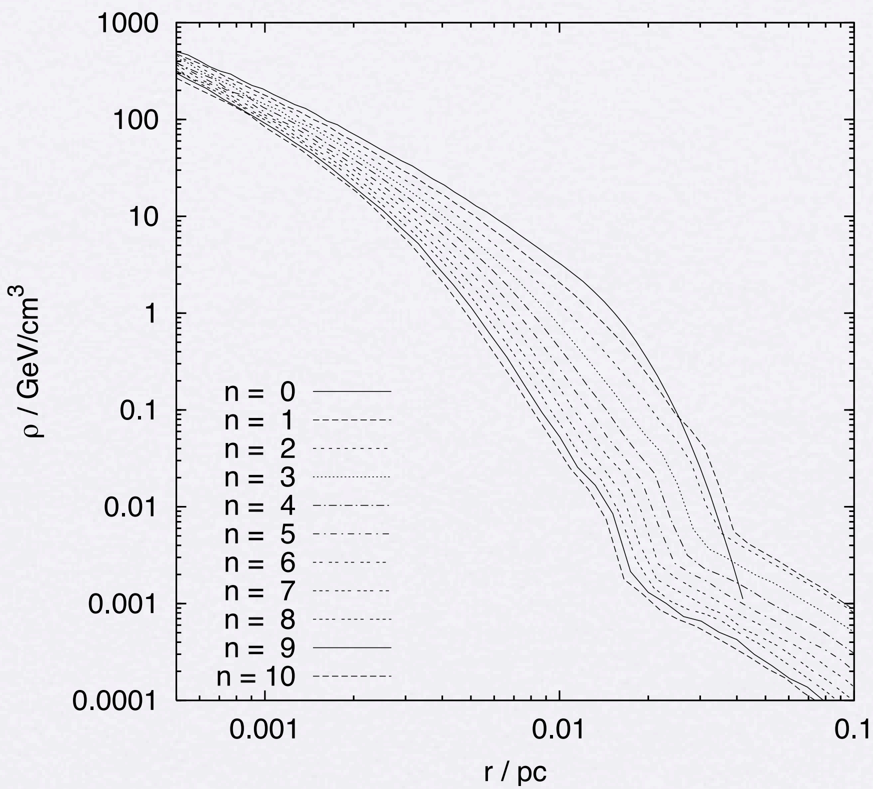
But micro-halo then undergoes a re-equilibriation process:

mass-loss is less than would naively be expected from energy-input.

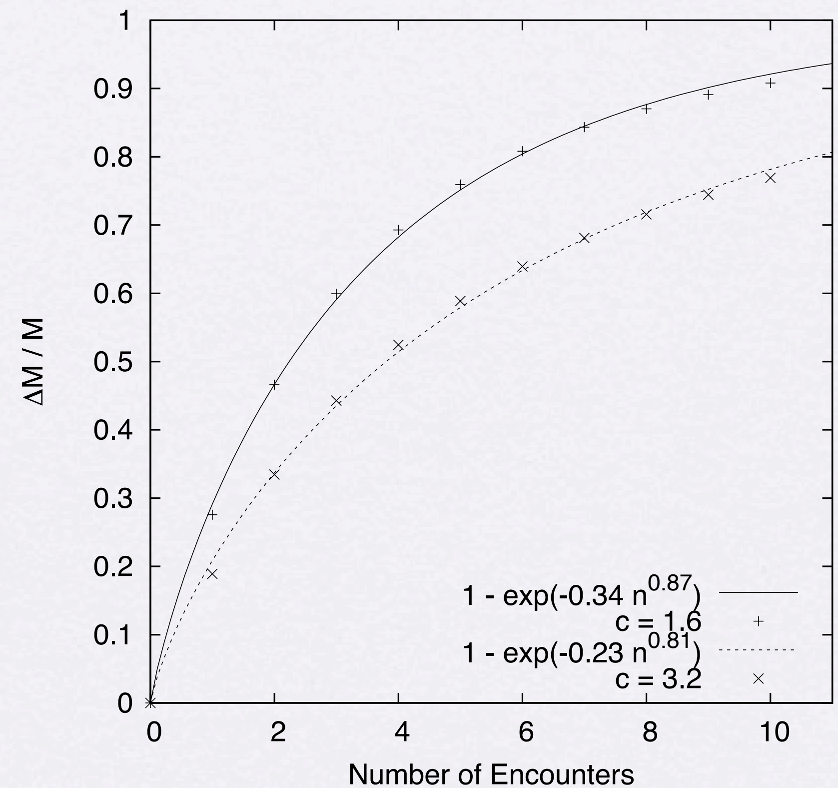
also need to take into account change in density profile when considering multiple interactions.

Effect of multiple encounters on

density profile



mass loss



Goerdt et al.

Bottom line (for stellar encounters)

Earth mass micro-halos in the solar neighbourhood will typically lose most of their mass on a time scale of order the age of the MW.

BUT

Even if most of mass is lost inner high density ‘cusp’ can remain relatively intact.

For individual micro-halos, mass loss depends on orbit (in particular stellar distribution along orbit) and initial density contrast.

(slightly?) more massive micro-halos can retain most of their mass.

Ideally want a single unified treatment of all the relevant dynamical processes (including distribution of micro-halo masses and size of fluctuations from which micro-halos form). Is this tractable?......

See talks in DM distribution and indirect detection parallel session for work in progress in this direction.

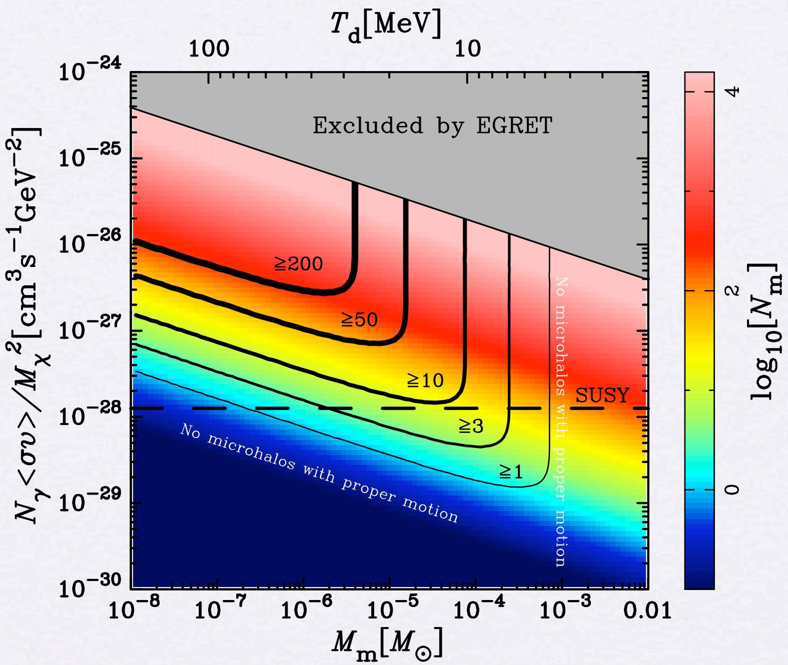
Implications

Indirect detection

i) Individual micro-halos

[Diemand, Moore & Stadel; Moore, Diemand,Stadel & Quinn; Koushiappas]

If (the dense central regions of) a few per-cent of the micro-halos survive heirarchical structure formation, there will by numerous micro-halos within ~pc, which will potentially be detectable by GLAST as high proper motion sources.



Koushiappas

Dependence of number of micro-halos with detectable proper motion on cut-off mass and annihilation cross-section.

Assumes 0.2% of local mass in micro-halos (& micro-halos have a NFW profile with concentration c~1).

But Pieri, Bertone & Branchini consider various assumptions for c(M), find EGRET limits on diffuse gamma-ray background rule out scenarios with detectable micro-halos.

How reliable are extrapolations from larger scale simulations?
Need to take into account micro-halo mass-loss/profile change.

ii) Enhancement of diffuse flux

[Lots of papers by lots of people]

$$\frac{\mathrm{d}n}{\mathrm{d}m} \propto m^{-\alpha} \qquad \alpha \sim 2$$

If this substructure mass function holds for all scales down to cut-off, equal mass per subhalo mass decade -> approximately equal constant contribution to annihilation luminosity.

Exact mass dependence of sub-halo contribution to flux depends on how density profiles (in particular concentration) scale with mass.

Some recent results:

Diemand, Kuhlen & Madau:

Via Lactea (highest resolution simulation to date of a Milky Way like halo).
Substructure increases annihilation luminosity by ~40%
Expect this would increase substantially (to a factor of a few) with increased resolution.

Pieri, Bertone & Branchini; Colafrancesco, Profumo & Ullio

Small mass subhalos provide biggest contribution to diffuse flux.

See DM distribution and indirect detection parallel session.

Direct detection

Probes WIMP density & velocity distribution in local sub-milli-pc region.

Probability of being within surviving central regions of micro-halo tiny......

How is the material removed from the micro-halos distributed?

i) filling factor of streams small, local dm density zero
[direct detection impossible]

ii) local dm dist consists of a small number of discrete streams
[detailed signals (energy spectrum, time & direction dependence) depend on velocities and densities of streams,
measuring WIMP mass and cross-section impossible]

iii) streams well mixed, local dm dist essentially smooth
['standard' calculations of energy spectrum (& exclusion limits) probably reasonable approx,
time and direction dependence will depend on exact velocity dist]

Some calculations/estimates:

Helmi, White & Springel

estimate > 6500 streams in solar neighbourhood

density in stream varies as $(t/t_{orb})^{-3}$ (mixing depends on range of orbital frequencies)

normalise to (scaled) simulation of MW

Stiff & Widrow

local DM dist non-smooth

'non-cosmological' simulation, reversed and rerun with more particles in regions that end up in solar neigbourhood

large softening required to suppress chaos and allow reversibility

No definitive answer yet.

non-WIMPy DM candidates

Warm DM (e.g. keV sterile neutrinos)

[e.g. Abazajian] power spectrum suppressed for k> O(1/kpc).

MeV DM

[Hooper, Kaplinghat, Strigari & Zurek] remains in kinetic equilibrium with cosmic neutrino background until T ~ 2 keV, power spectrum truncated at free-streaming scale ~ 2 kpc ($\equiv M \sim 10^7 M_\odot$).

Axions

To have a cosmologically interesting density axions must be produced non-thermally (mis-aligment angle, emission by axionic strings).

If inflation doesn't occur, or re-heat temperature above Peccei-Quinn scale, large spatial fluctuations in value of axion field (and hence axion density) on horizon scale at QCD phase transition.
First axion halos form around matter-radiation equality: $M \sim 10^{-12} M_\odot$
[Hogan & Rees; Kolb & Tkachev; Zurek, Hogan & Quinn]

present day axion distribution?

Summary

- WIMP direct and indirect detection probe the dark matter distribution on small scales.
- Collisional damping and free-streaming erase density perturbations on small scales and set the scale of the first halos to form [O($M \sim 10^{-6} M_{\odot}$) but with a range of several orders of magnitude depending on WIMP-lepton scattering cross-section].
- Do (a significant fraction of) these halos retain a significant fraction of their mass?
- And how is the material which is lost distributed?