Hierarchical formation of the Milky Way with micro-solar mass resolution

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OUTLINE:

- √ Introduction: Mass Function of dark matter halos and substructure population;
- √ Spherical Collapse Montecarlo Mergertree with micro solar-mass resolution;
- √ Analytical estimate of the substructure mass function.

Introduction: DM halo mass function and substructures population

Density Field and DM halos:

Matter density fluctuations field at position x,

$$\delta_i(x) = \frac{\rho(x) - \rho_b}{\rho_b}$$

ρ_b represents the background density of the universe

The fluctuation, after expanding with the universe, will collapse and form a virialized DM "M-halo" if the density field smoothed on scale R ($R^3 = 3M/4\pi\,\rho_b$) has a value larger than that predicted by the spherical collapse model.

For ΛCDM universes and at the present time, this value $324\,\rho_b$ and is independent on M.

Power Spectrum

M-halo mass variance

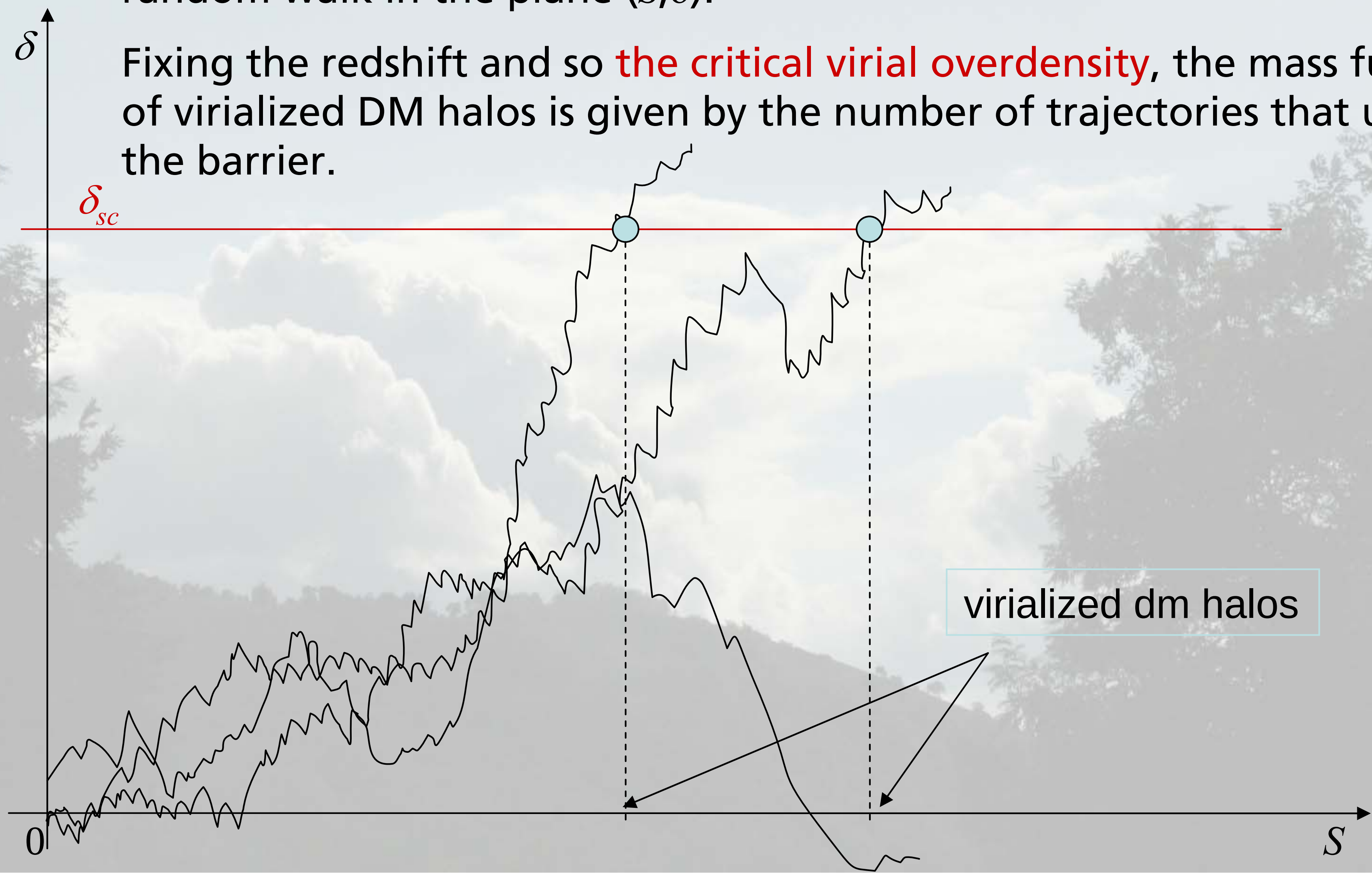
$$\sigma_M^2 = S = \frac{1}{(2\pi)^3}\int dk^3 P(k) W^2(kR)$$

Spherical Collapse Mass Function:

Press & Schechter 1974, Bond et. al 1991, Lacey & Cole 1993

In the excursion set model a density fluctuation is represented by a random walk in the plane (S,δ).

Fixing the redshift and so the critical virial overdensity, the mass function of virialized DM halos is given by the number of trajectories that upcross the barrier.

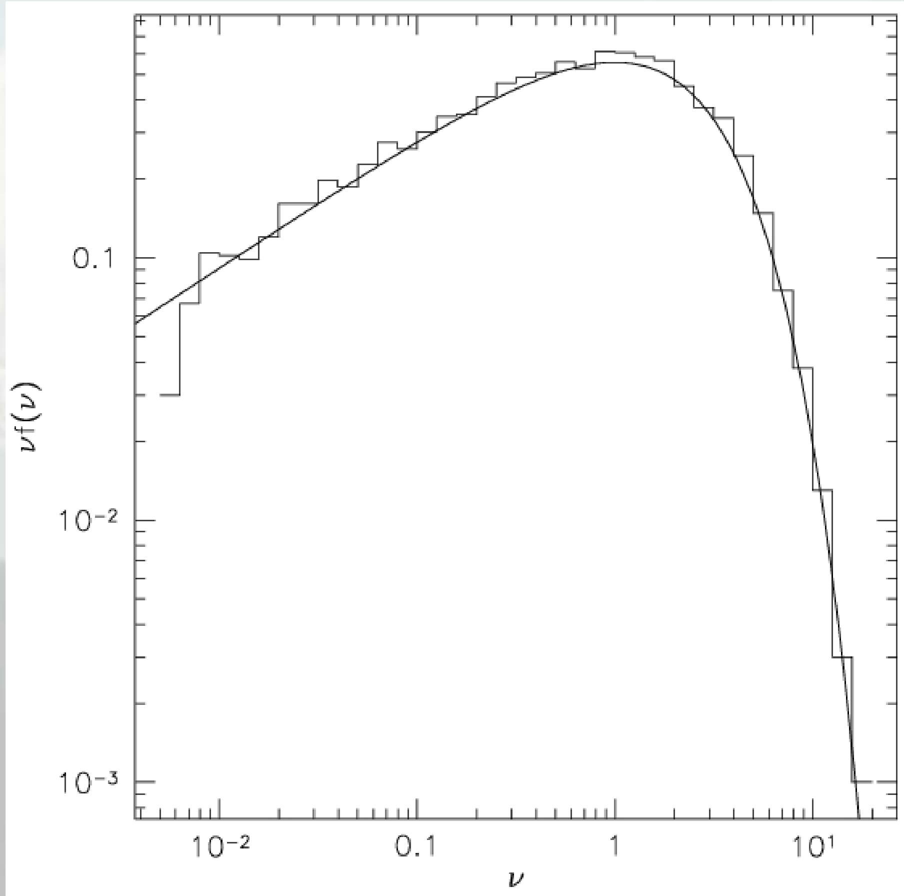


Spherical Collapse Mass Function:

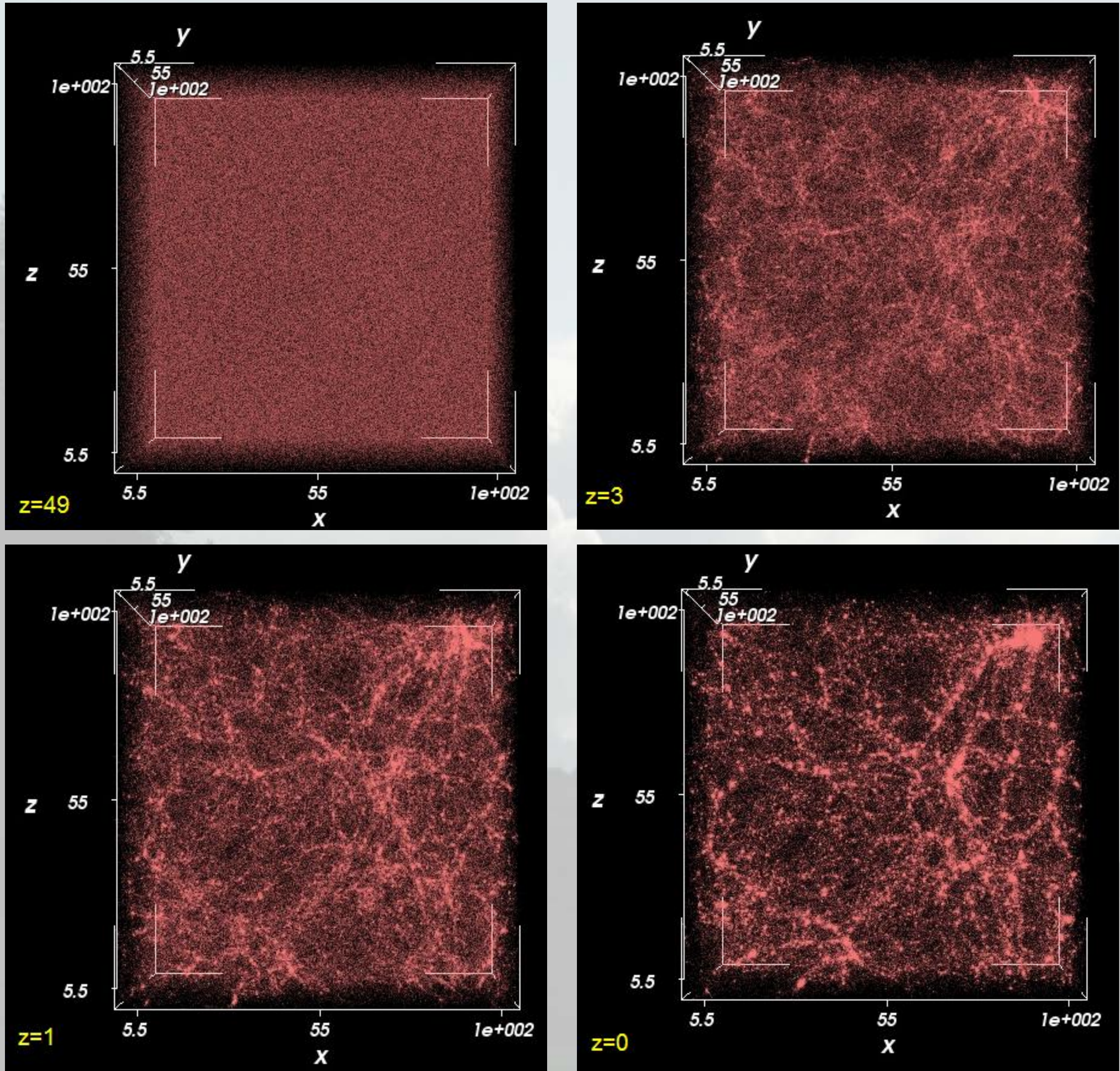
$$f_{SC}(\nu)d\nu = \sqrt{\frac{\nu}{2\pi}}\exp(-\nu/2)\frac{d\nu}{\nu}$$

universal mass function

$$\nu = \frac{\delta_{sc}^2}{s}$$

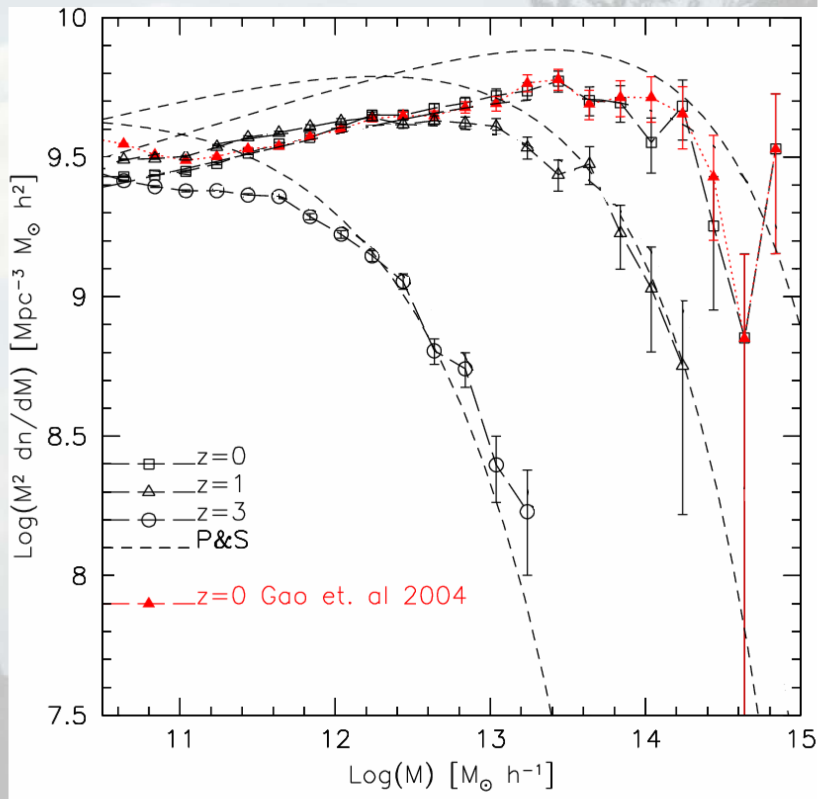


Comparison with N-Body Simulations:



The collapse of DM halos is not spherical but ellipsoidal.

White & Silk 1979, Peebles 1980, Lemson 1993…

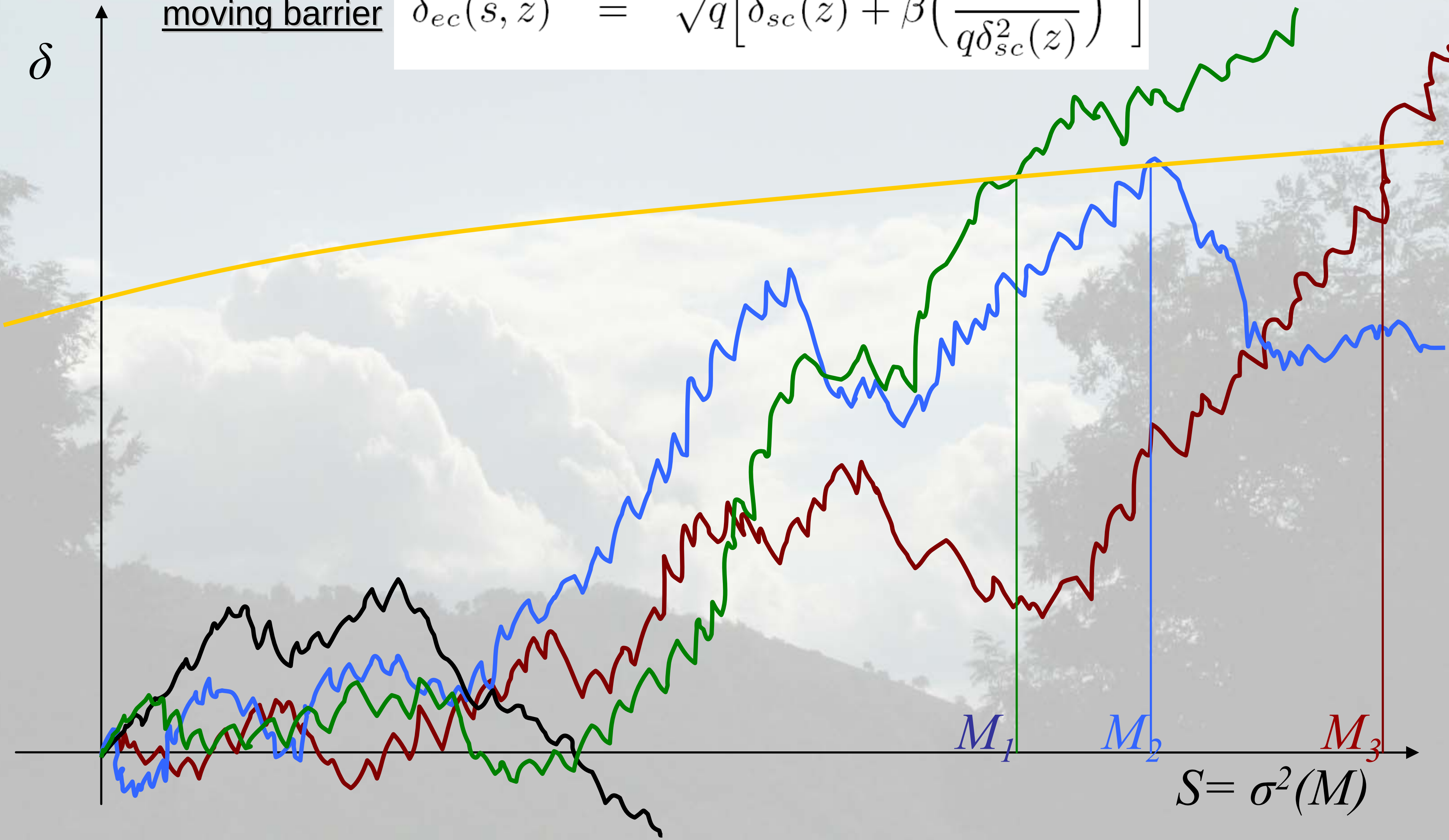


GIF2: Gao et al 2004

Ellipsoidal Collapse:

Sheth, Mo & Tormen 2001, Sheth & Tormen 2002

moving barrier
$$\delta_{ec}(s,z) = \sqrt{q}\left[\delta_{sc}(z) + \beta\left(\frac{s}{q\delta_{sc}^2(z)}\right)^{\gamma}\right]$$



Ellipsoidal Collapse:

$$\delta_{ec}(s,z) \quad = \quad \sqrt{q}\left[\delta_{sc}(z) + \beta\left(\frac{s}{q\delta_{sc}^2(z)}\right)^{\gamma}\right]$$

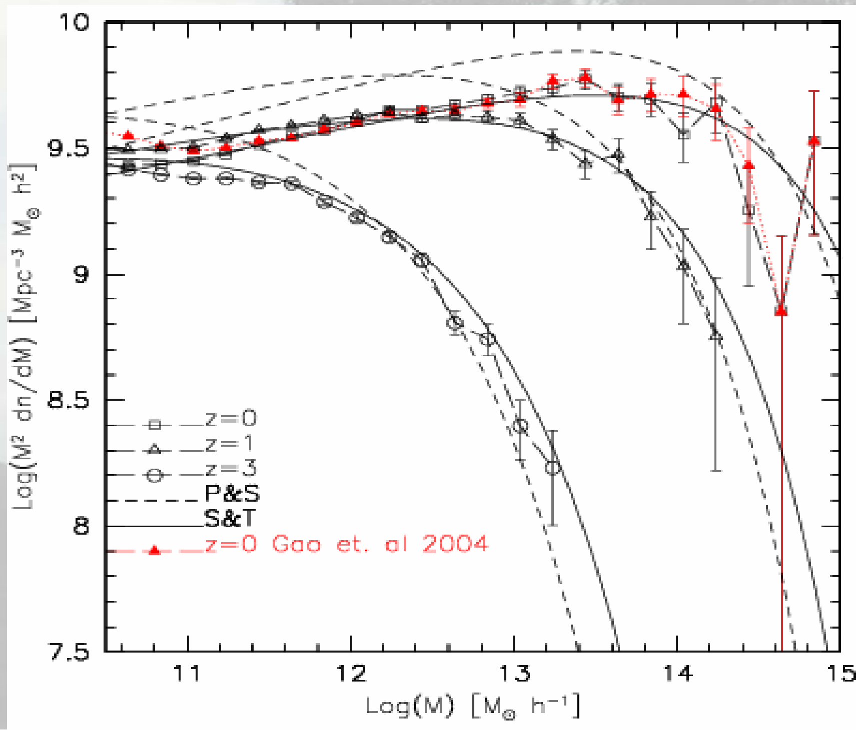
β and γ are motivated by an analysis of the collapse of homogeneous ellipsoids, whereas the value of q comes from requiring that the predicted halo abundances match those seen in simulations (Sheth, Mo & Tormen 2001 – Sheth & Tormen 2002).

q$=0.707$, $\beta=0.5$ and $\gamma=0.6$

probability of first upcrossing distribution

$$f_{ST02}(s)ds = \frac{|T(s)|}{\sqrt{2\pi s}}\exp\left[-\frac{\delta_{ec}^{\ 2}}{2s}\right]\frac{ds}{s}$$

for $\beta=0$ and $q=1$: $\quad f_{ST02}(s)ds \rightarrow f_{SC}(s)ds$



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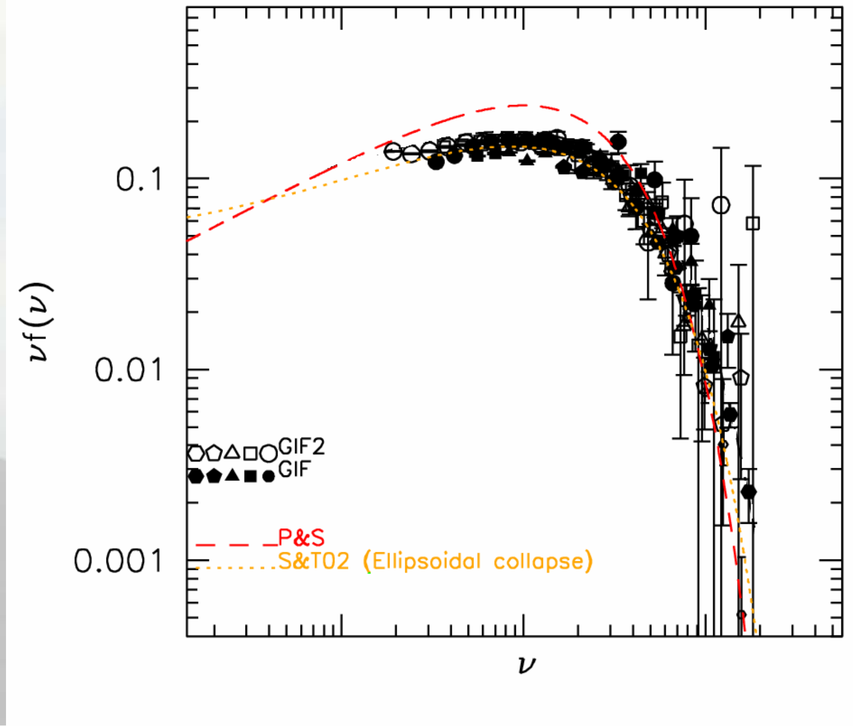

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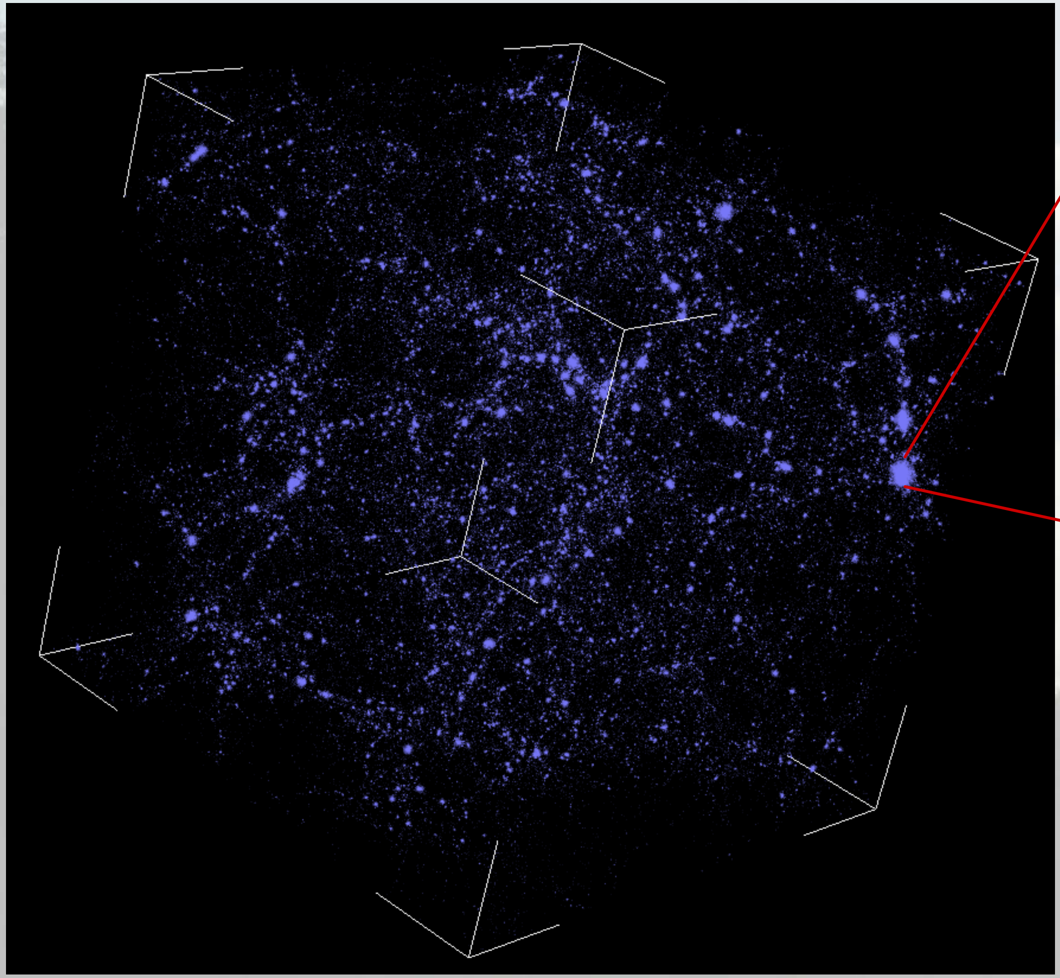
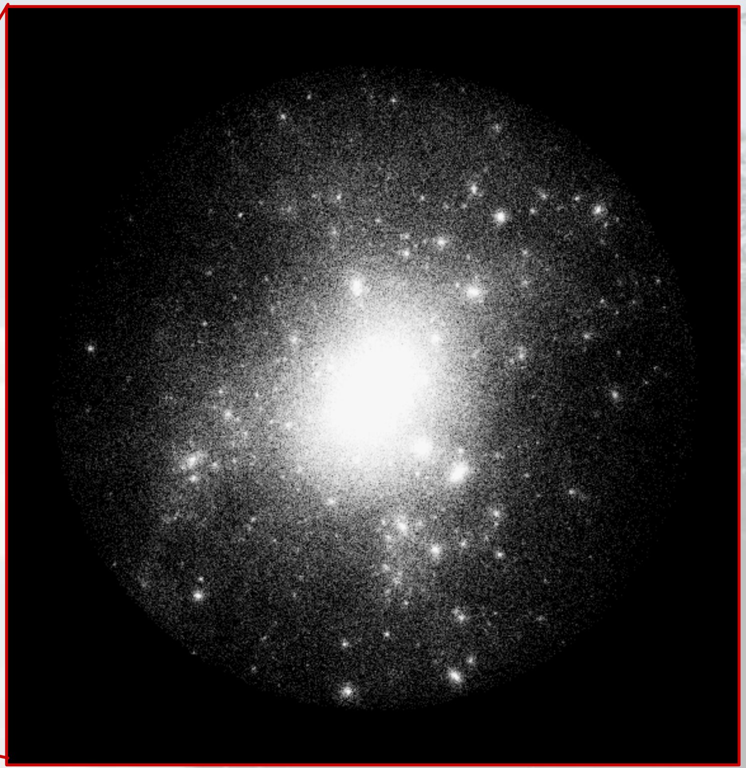
DM halos population: substructures

DM particles distribution in DM halos at z=0;
GIF2 simulation: N=400^3; $L=110$ Mpc/h





The dark matter halos are populated by substructures. The core of progenitors accreted along the merger-history-tree can still survive in the main progenitor halo at the present day.

Substructures Population:

Substructures Mass Function in N-Body simulations:

- there is no unique way to identify the clumps - different codes have been developed and are available in the literature;

- the mass function is limited in resolution – if m_{dm} is the particle mass, the smallest substructure must have at least few tens of DM particles;

SKID (Stadel 1998)	Particles are moved along a local density gradient, and then grouped by FOF, followed by gravitational unbinding. (derived from DENMAX, Gelb & Bertschinger 1994)
HFOF (Gottloeber et al. 1999)	Plain FOF is applied with a hierarchy of linking lengths
BDM (Klypin et al. 1999)	Local maxima in the density are identified (somehow), and then the bound set of particles in spherical apertures is determined
HOP (Eisenstein & Hut 1998)	A local density estimate is computed, and then particles are attached to their nearest neighbours. A set of rules connects and prunes the isolated groups.
SUBFIND (Springel et al. 2001)	Based on local density estimates, topological criteria are used to find isolated overdense regions which are then subjected to a gravitational unbinding procedure
MHF (Gill, Knebe & Gibson 2004)	An adaptive grid is used to locate density maxima. Around each maximum, a spherical aperture is grown until an upturn in the spherical density profile is detected. This is followed by gravitational unbinding and removal of subhalo duplicates.
SURV (Tormen & C.G.)	The substructures are defined as self-bound groups of particles belonging to halos accreted along all the branches of the tree of a present day halo.

Substructures Population:

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Substructures Population:

The purpose of this talk is to describe two methods (two different definitions of substructures) to populate a Milky-Way size halo with substructures until the $M_{j,dm}=10^{-6}\ \mathrm{M}_{sun}$.

No tidal stripping and merging among the subhalos

Neutralino Jeans mass

Spherical Collapse Montecarlo Mergertree with micro-solar mass resolution:

mass accreted along the main branch of the tree

subhalos[1]

Giocoli, Pieri, Tormen et al. in preparation

Progenitor Mass Function:

z ➔ *0*

Mass function of dark matter halos at redshift z, that will end up at redshift $z=0$ in a M-halo. We consider $M=10^{12}\mathrm{M}_{sun}/h$ – Milky Way size dark matter halos.

$$f(m,z\,|\,M)dm = f(s,\delta_1\,|\,S,\delta_0)ds$$

0 ➔ *z*

Mass function of dark matter halos in which a present-day M-halo is split: idea behind the Montecarlo Merger-History-Tree.

According to the spherical collapse model the progenitor mass function has a Gaussian distribution in term of ν:

$$\nu = \frac{(\delta_1 - \delta_0)^2}{s - S} \qquad f_{SC}(\nu)d\nu = \sqrt{\frac{\nu}{2\pi}}\exp(-\nu/2)\frac{d\nu}{\nu}$$

The method: progenitors mass function at redshift δ_1

Sheth & Lemson 1999

White Noise Gaussian initial condition (disconnected volumes): $S \sim 1/M$

In the Poissonian case an M-halo, at the present time, occupies a volume V_M where:

$$\frac{M}{V_M} = \bar{\rho}(1 + \delta_0)$$

Extract ν from a Gaussian distribution and compute s so m,

$R = M\text{-}m$

$$\frac{m}{V_m} = \bar{\rho}(1 + \delta_1)$$

Preserving the mass and the volume, from the previous equations we have the overdensity of the remaining mass R:

$$\delta_R = \delta_0 - \frac{(\delta_1 - \delta_0)}{R / M}$$

The method: progenitor mass function at redshift δ_1

<u>*White Noise Gaussian initial condition (disconnected volumes)*</u>: $S\sim 1/M$

Define: M, δ_0, δ_1, $R=M$ and $D = \delta_1\text{-}\delta_0$:

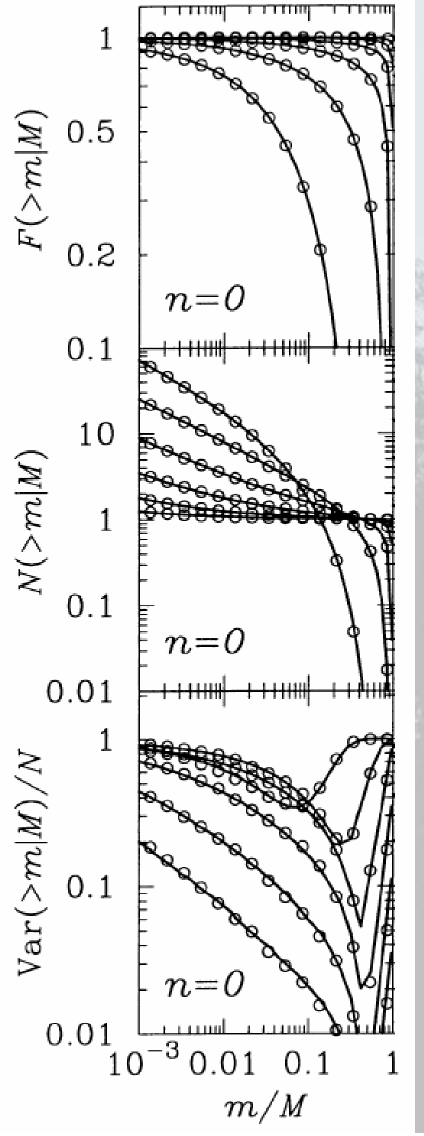
DO

- draw a mass from $f(s|S(R),D)$
- $R = M - m$
- $D = \delta_1\text{-}\delta_R$

(interate until R is as small as desired)

END DO

The progenitors mass function obtained match the spherical collapse prediction at each redshift (it is time step independent), this method can split a M-halo down to desired mass resolution R.



$(z_1 - z_0) = 0.01.\ 0.03,\ 0.1,\ 0.3,\ 1$ and 3.

The method: progenitors mass function at redshift δ_1

<u>general initial condition</u>: $S=g(M)$

As density fluctuations become larger, they feel the gravity of surrounding matter: overdense systems tend to accumulate more matter. ➔ *<u>the volumes are not mutually independent</u>*.

Extending the mergertree to generic models ($n=-1,-2$ and CDM) it is necessary to underline that when expressed as function of the variance rather than the mass. All excursion set quantities are independent of power spectrum.

Define: M, δ_0, δ_1, $R=M$ and $D = \delta_1$-δ_0:

DO

- draw a mass from $f(s|S(R),D)$
- m, $m_{wn,}$ $\eta=m_{wn}/m$ (statistic weight)
- $R = M - \eta\, m$ or $R = M - m_{wn}$
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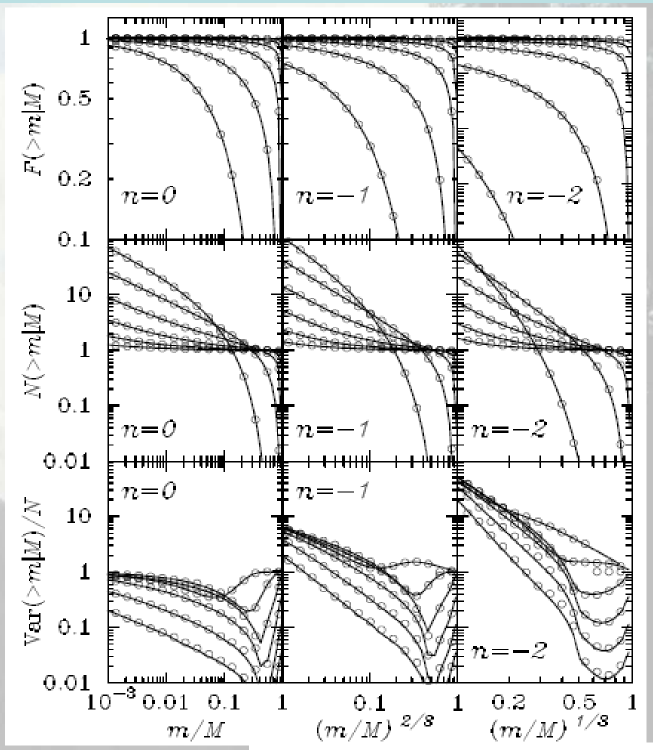

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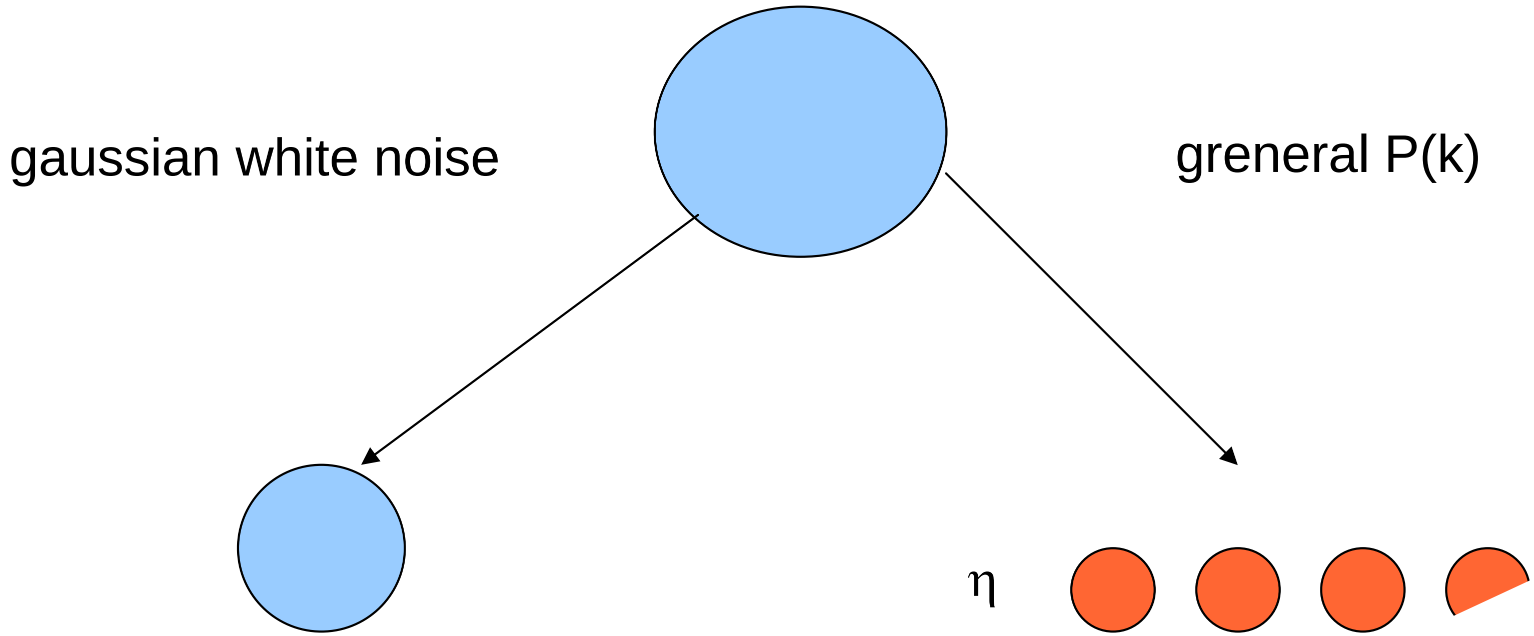
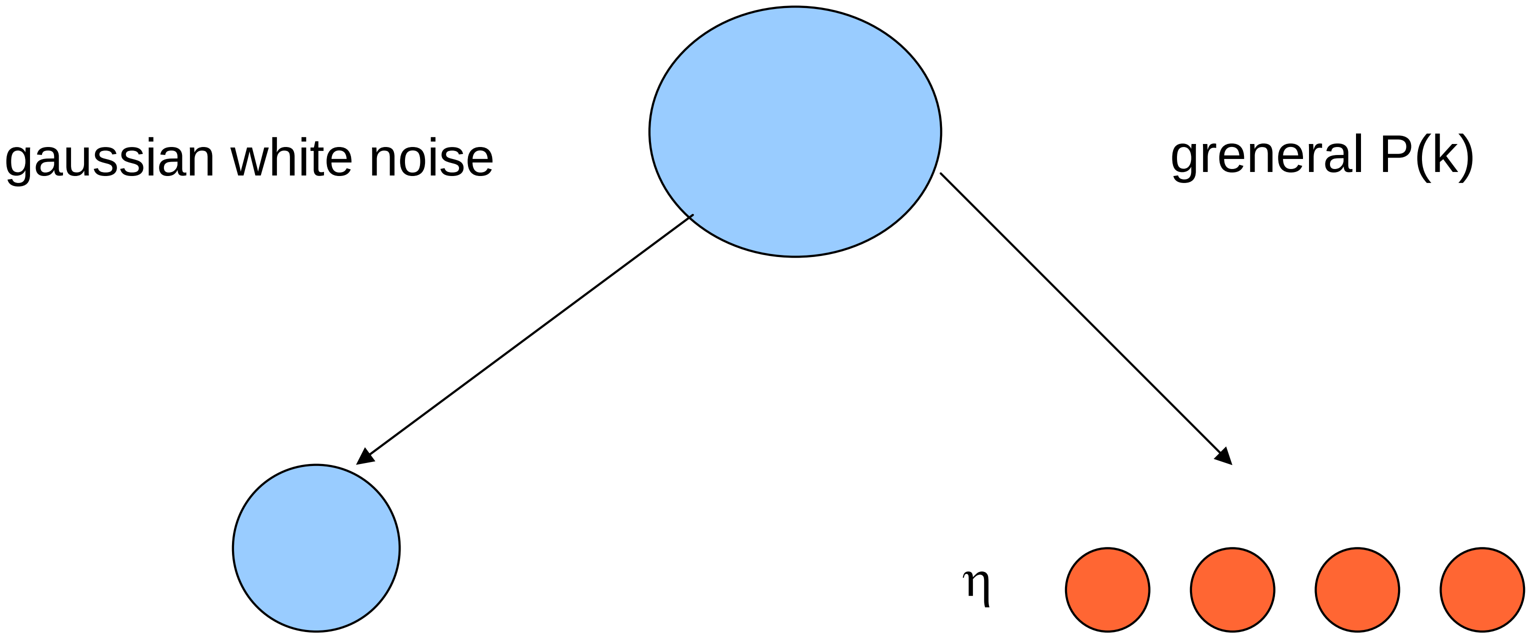

gaussian white noise
greneral P(k)
η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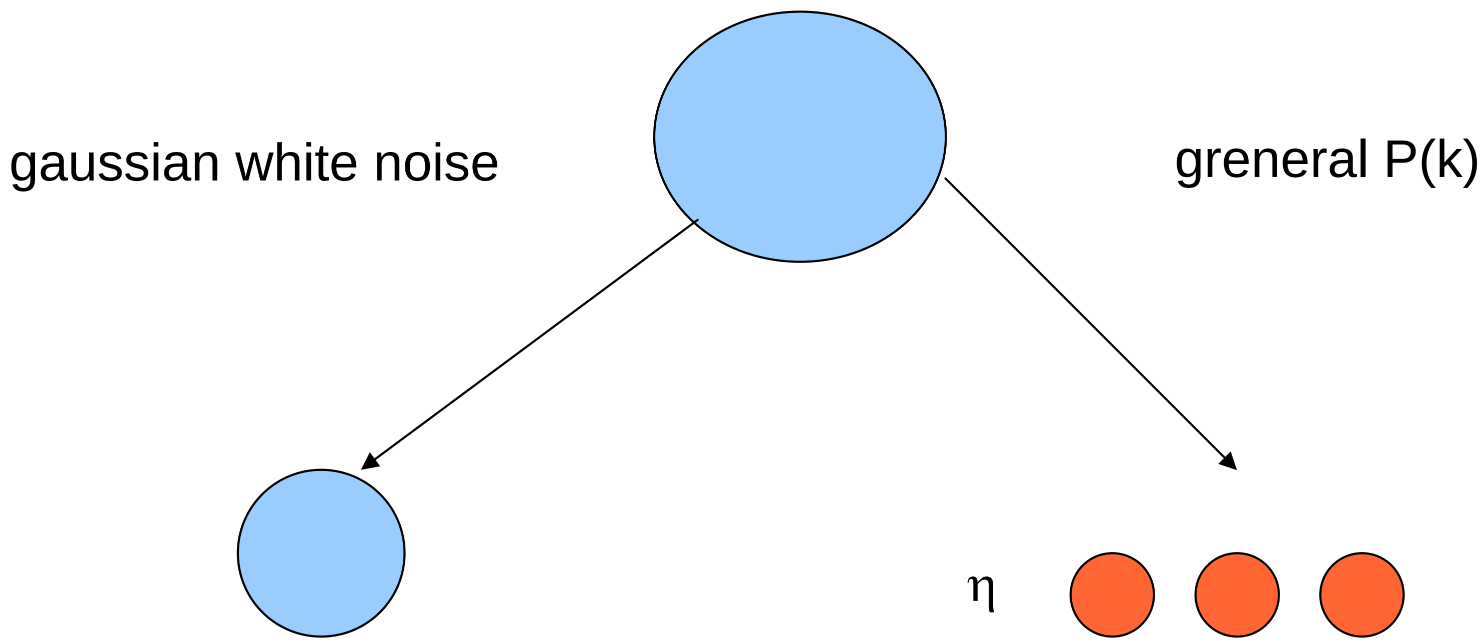
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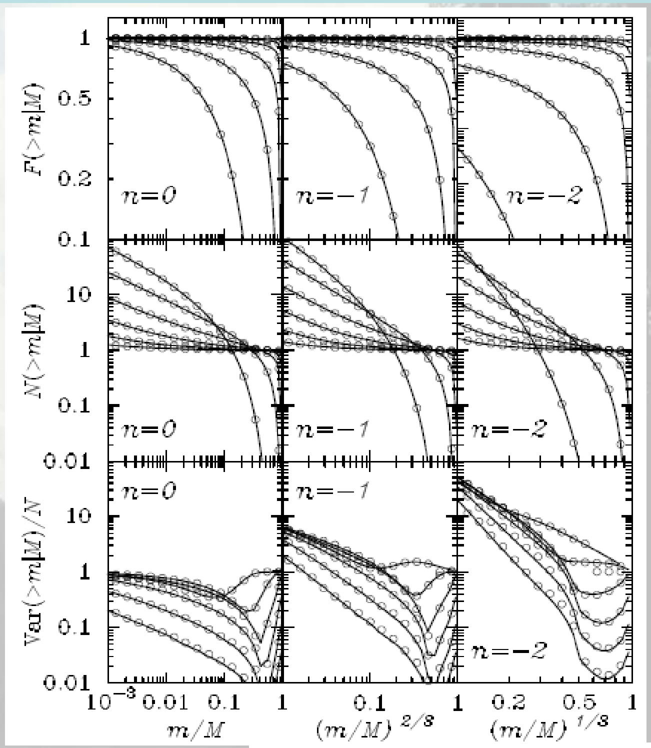
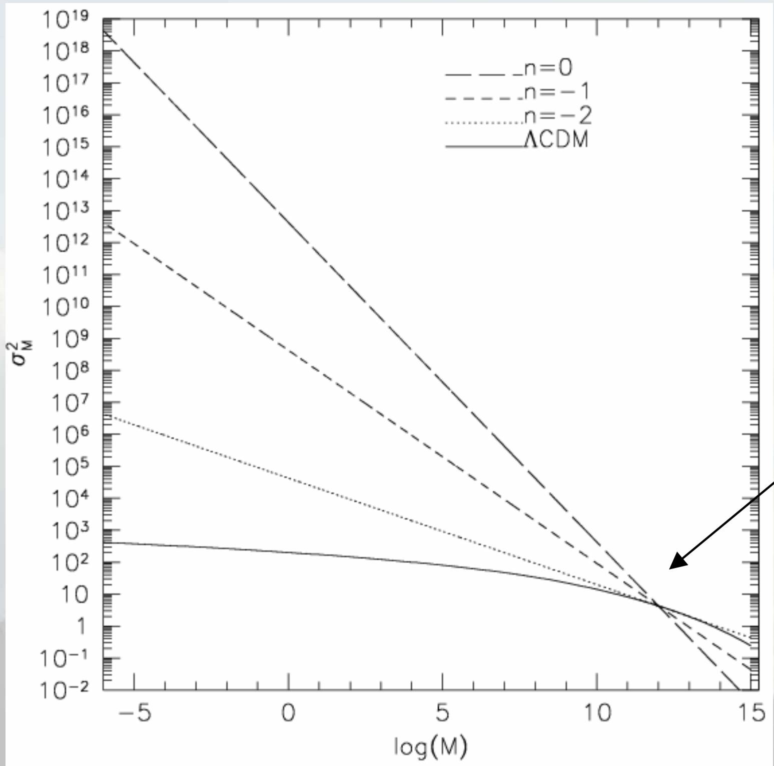


$(z_1 - z_0) = 0.01, 0.03, 0.1, 0.3, 1$ and 3.

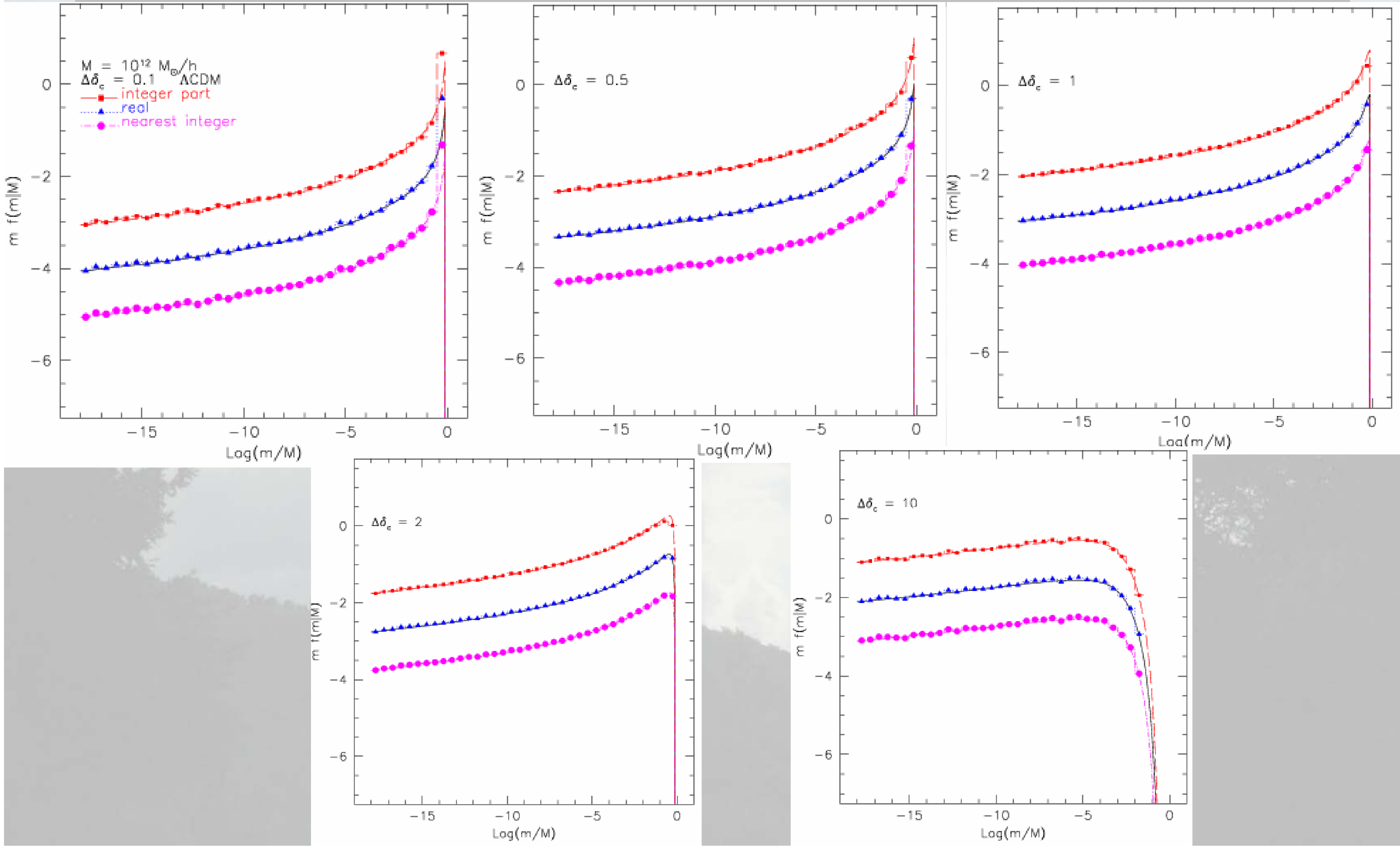
One single step partition of a Milky-Way size DM Halo:



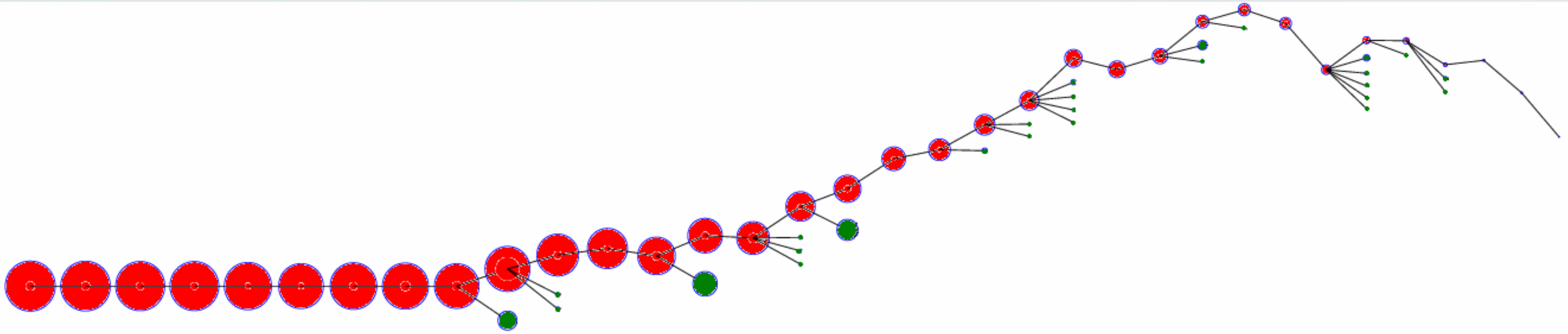
The Mass Variance of different power spectrum are normalized to a present-day Milky Way-size DM Halo in *CDM*.

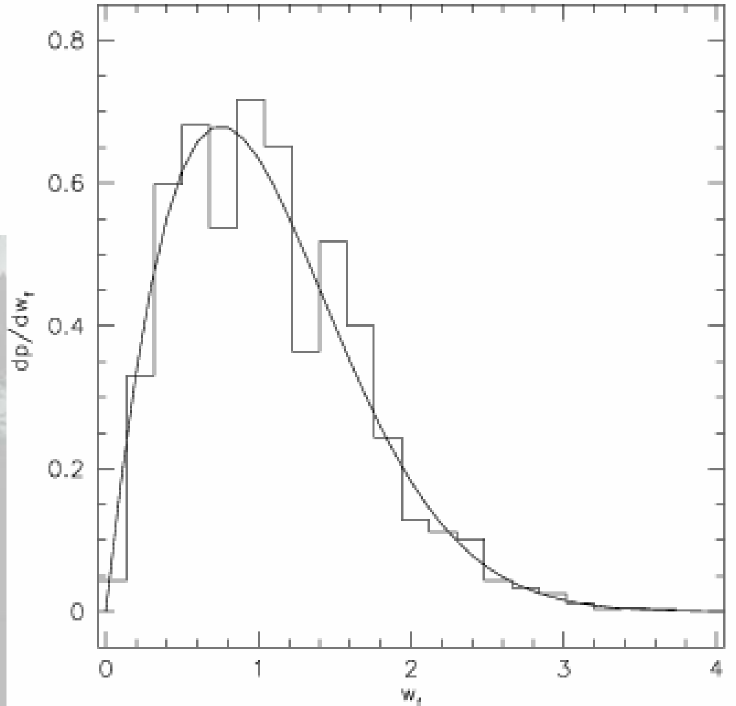
Ω_m=0.3, Ω_Λ=0.7, *h*=072, σ_8=0.7

One single step partition of a Milky-Way size DM Halo:



Possonian ($n=0$) Mergertree along the main branch:





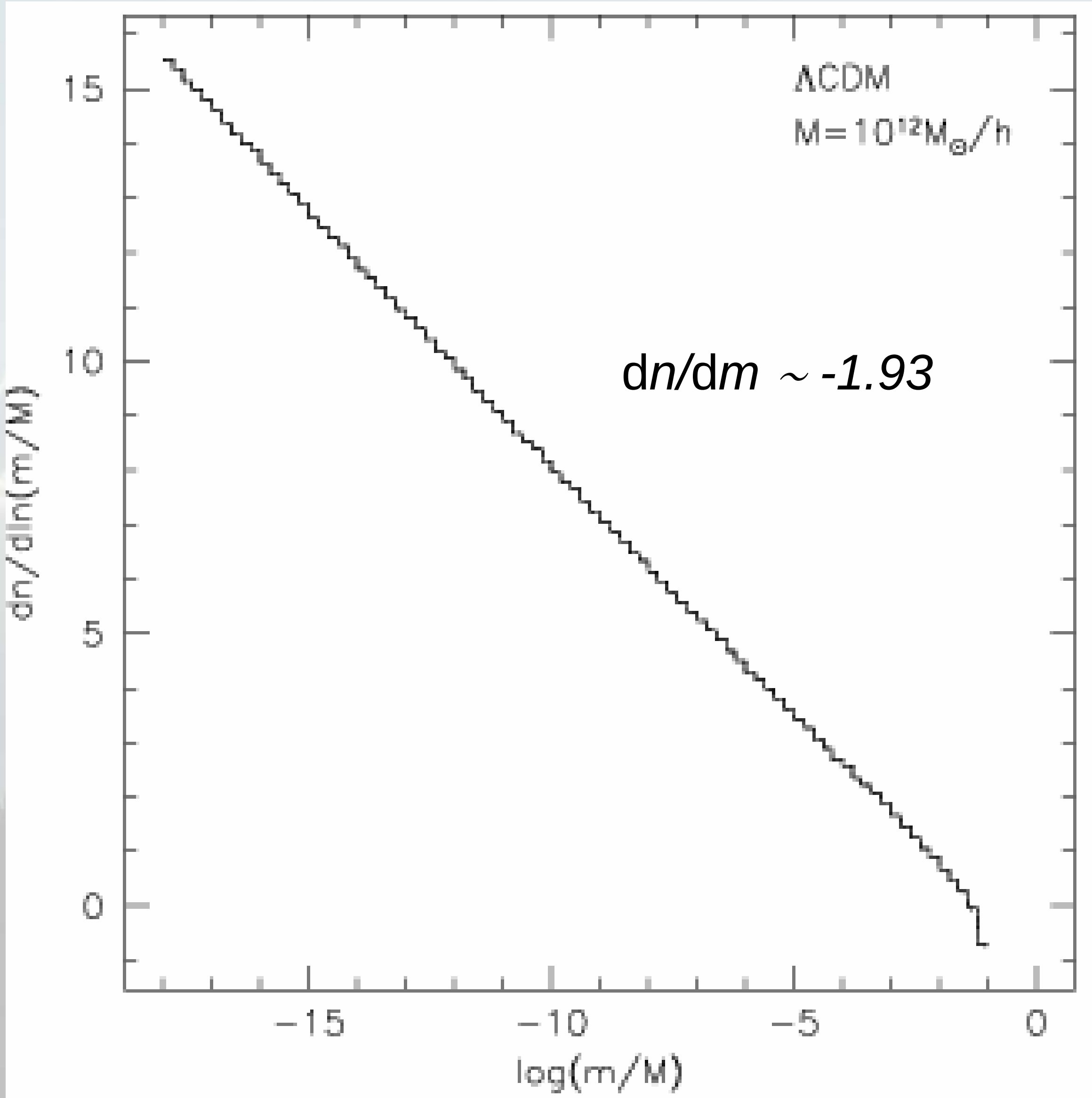
Lacey & Cole 1993 prediction:

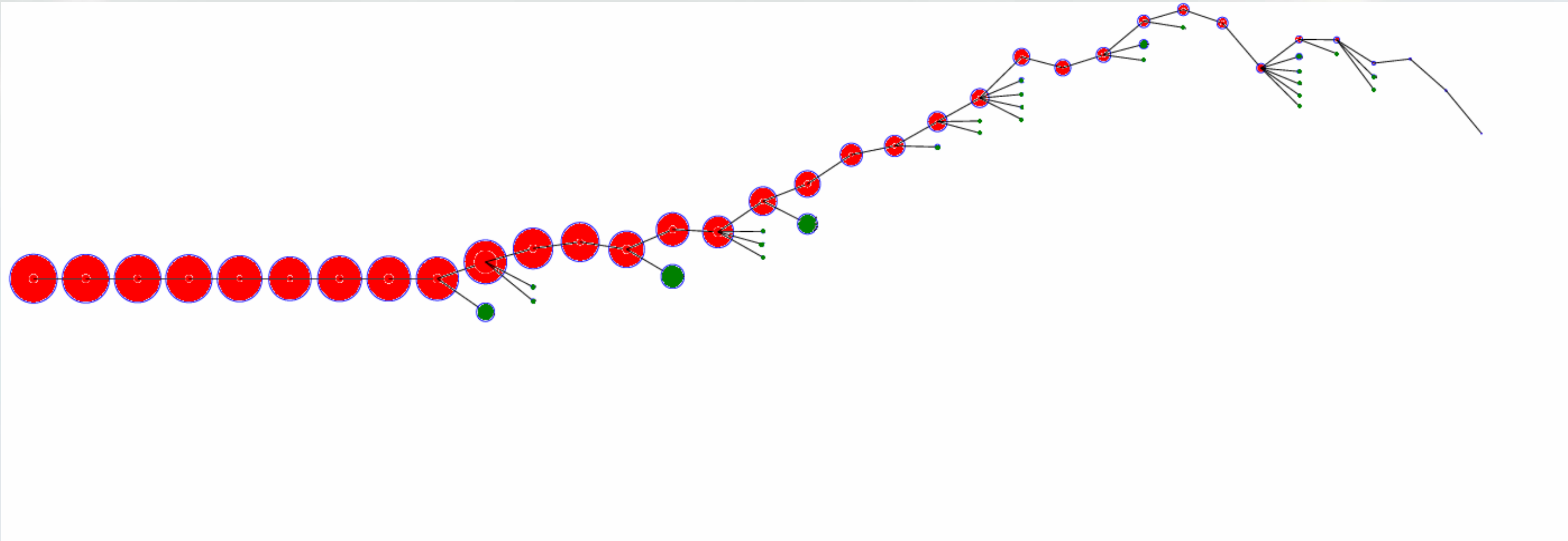
$$p(w_f) = 2w_f \, erfc(w_f / \sqrt{2})$$

$$w_f = \frac{\delta_f - \delta_0}{\sqrt{S(M/2) - S(M)}}$$

formation redshift distribution:
z_f such that $M(z_f) = M/2$.

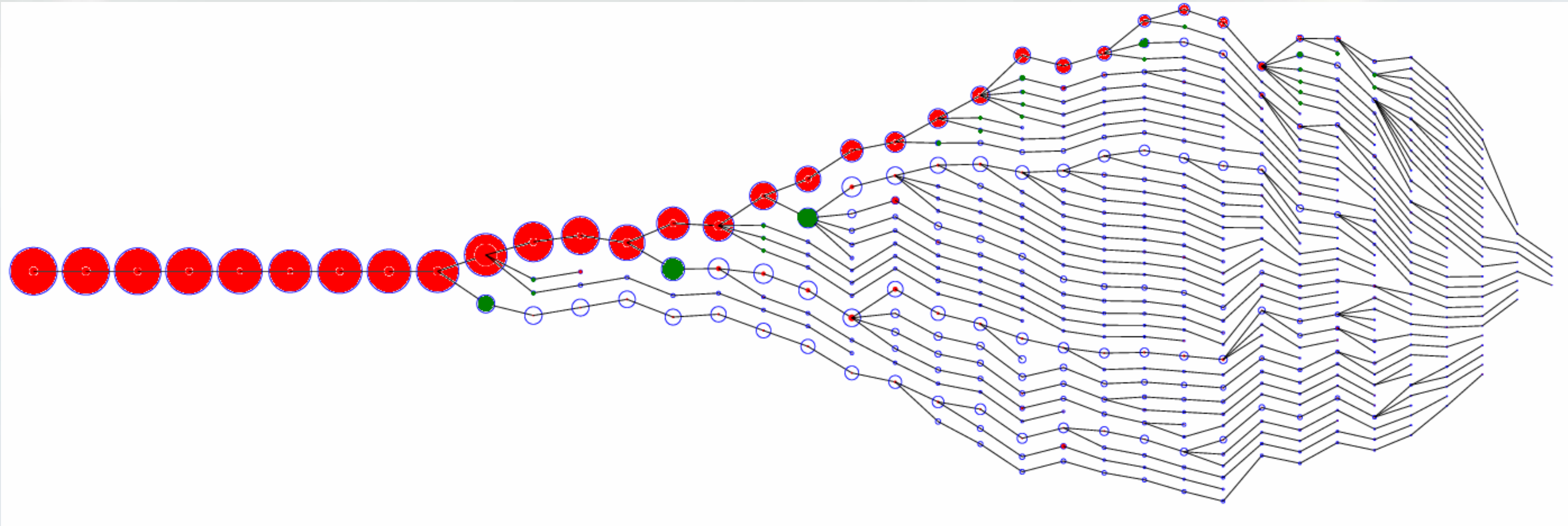
Mass accreted by the main branch:





Analytical estimate of the substructure mass function:

spherical and ellipsoidal collapse predictions



Analytical estimate of the substructure mass function:

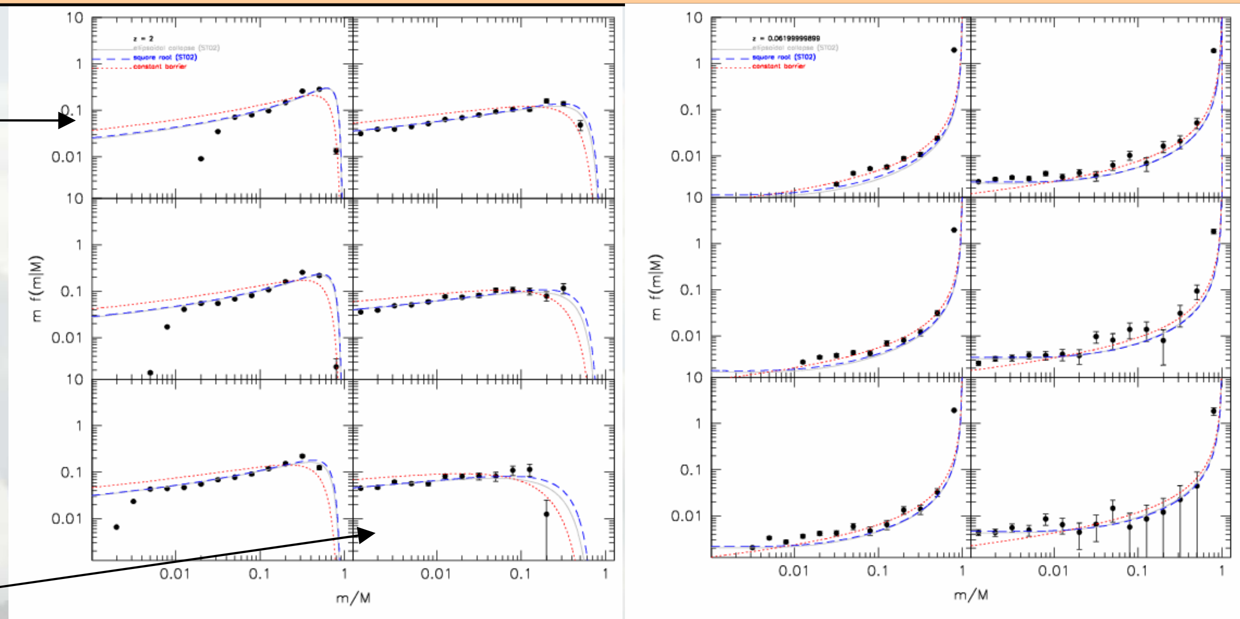
spherical and ellipsoidal collapse predictions

Giocoli, Pieri, Tormen in submition

Progenitor Mass Function:

Sheth & Tormen 2002 showed that a better fit to the conditional mass function in cosmological N-Body simulations is given by the probability of first upcrossing distribution of a moving barrier. However, fixing the initial redshift (δ_0) and mass (M and so S), the mass function is not self-similar as was in the spherical collapse case.

small present day halo masses.



large masses

$$f_{ST02}(s\,|\,S)ds = \frac{|\,T(s\,|\,S)\,|}{\sqrt{2\pi(s-S)}}\exp\left[-\frac{(\delta_{ec}-\delta_{ec,0})^2}{2(s-S)}\right]\frac{ds}{s-S}$$

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Progenitors of a present day M-halo:

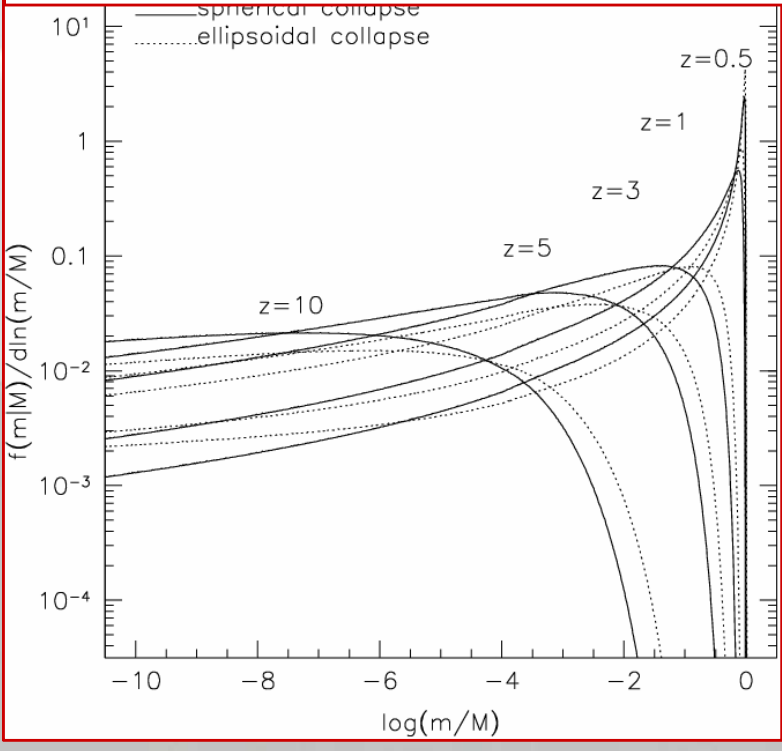
$$N(m, z \,|\, M)\,dm = \frac{M}{m} f(s \,|\, S)\,ds$$

$M = 10^{12}\ M_{sun}/h$

$M_{j,dmp} = 10^{-6}\ M_{sun}$

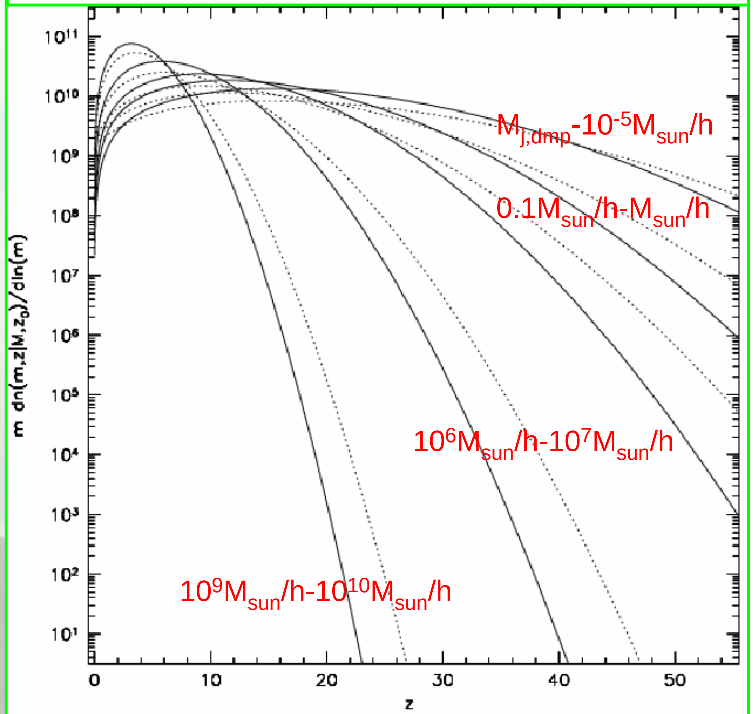
Average number of progenitors, at a given redshift,

in term of the progenitor halo mass.

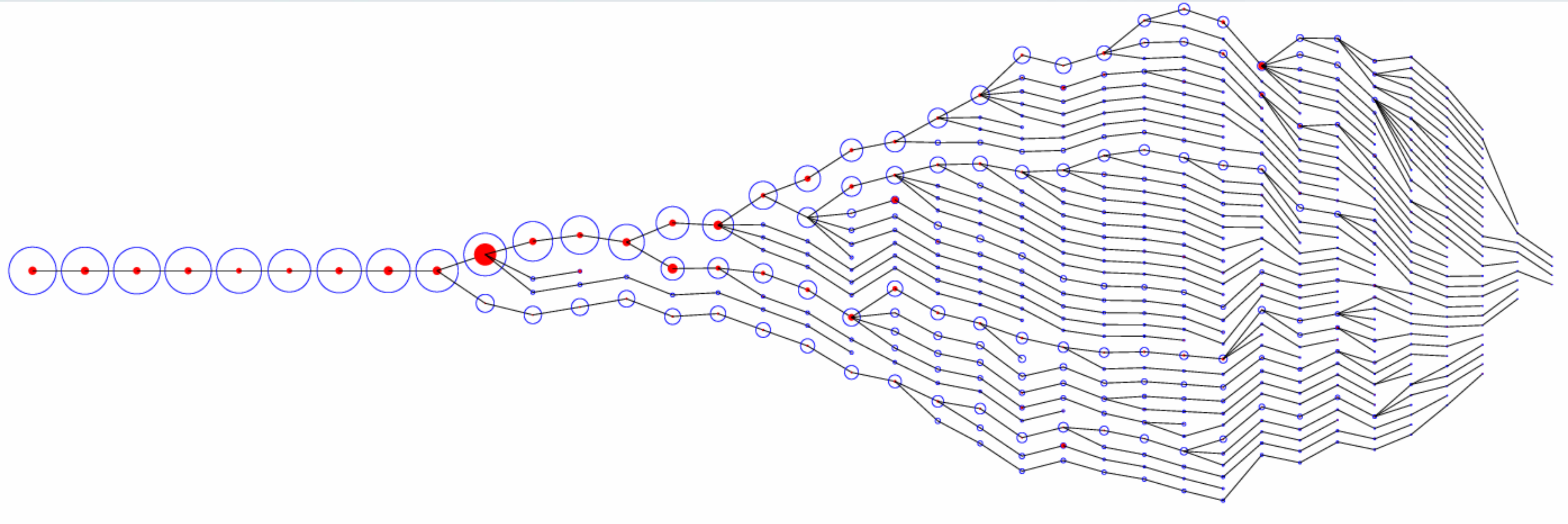


Average number of progenitors, in a given mass bin,

in term of redshift.



Substructures Population:



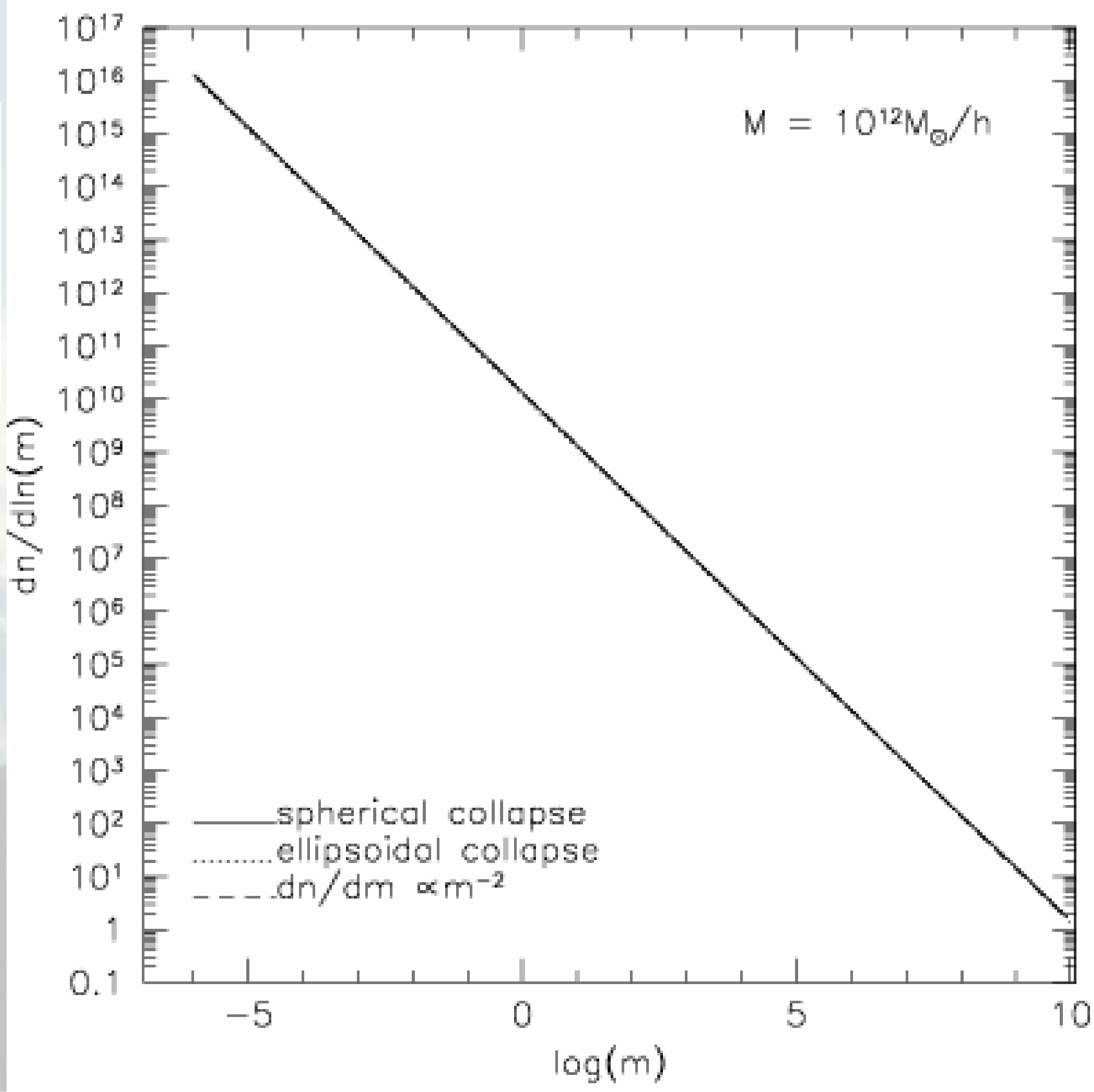
Integrating over redshift the average number of progenitors in a given mass bin, we obtain total mean number of progenitors.

However, in this kind of calculation we can count more than once the progenitors that will end up in the final system.

To overcome this problem we normalize the mass function of substructures assuming that for a Milky Way-size DM halo *10%* of its mass is in substructures with mass ranging from 10^7-$10^{10} \mathrm{M}_{sun}/h$.

Substructures Population:

This calculation assume no tidal stripping and merging between the progenitors halo. Each substructure preserves its virial mass.



-2,-1.7 is the slope found in N-Body simulations after tidal stripping and dynamical encounters between substructures.

Summary:

- Ellipsoidal collapse gives a more realistic description of the collapse of the dark matter halos as seen in N-Body simulations.

- Estimation of the mass accreted by the main branch of tree of a present-day Milky Way-size halo (-1.93).

- Looking at the all branches of the tree the substructures mass function goes like -2 (subhalos within subhalos…).

- *Gamma-Ray emission from these two distinct populations*.