The effect of energy amplification variance on the shock acceleration

Junichi Aoi

Yukawa Institute for Theoretical Physics

Kyoto university, Japan

Collaborators

Kohta Murase (YITP)

Shigehiro Nagataki (YITP)

Shock acceleration

Theme

Relativistic shock acceleration

Gamma-Ray Bursts

Shock acceleration is…

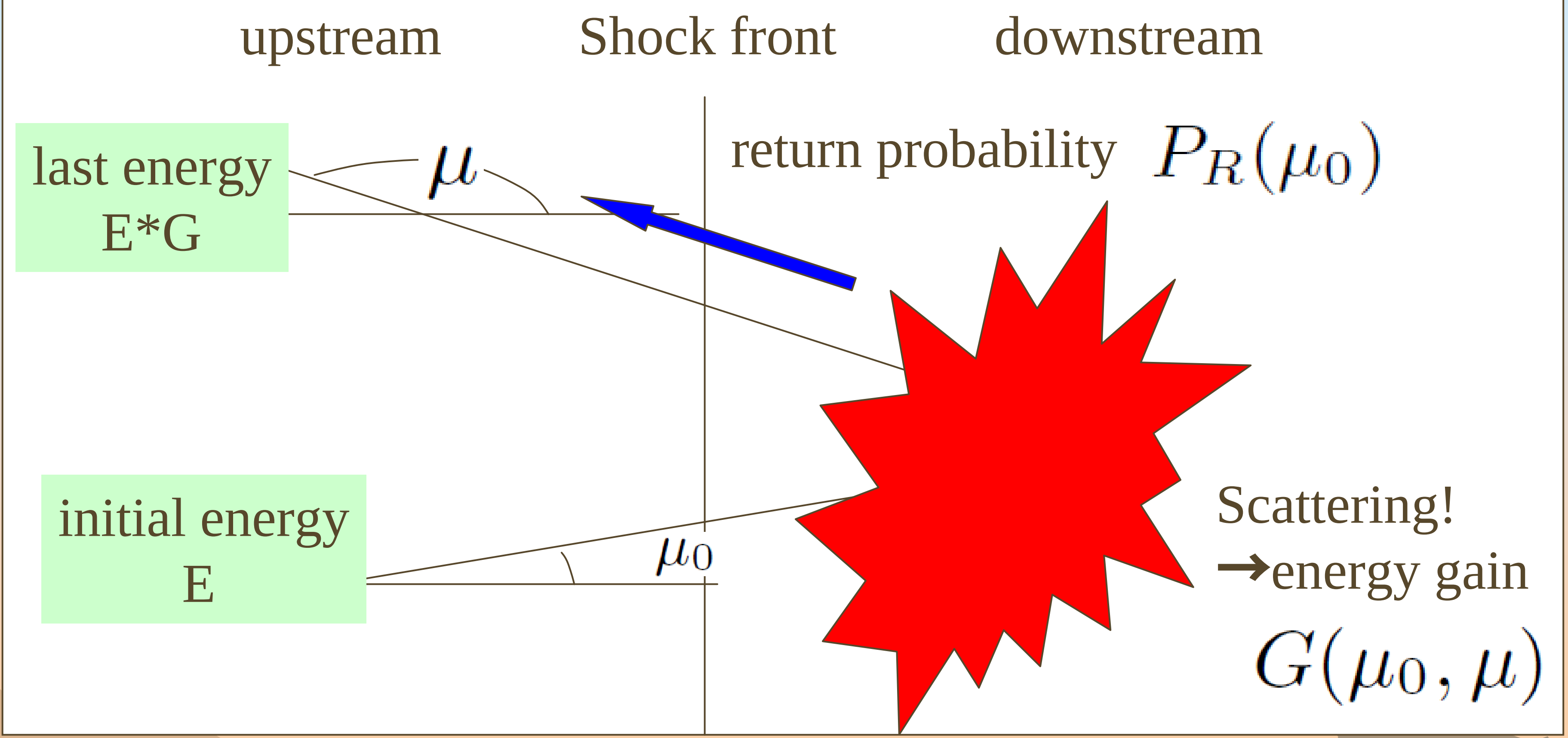
Particles are accelerated by gaining the energy from back ground plasmas through MHD waves. Particles are scattered by the MHD waves which are raised by instability of background plasmas.

Energy spectrum $$f(E)d^3\boldsymbol{p} \propto E^{-s}d^3\boldsymbol{p}$$

We examined the shock acceleration mechanism which is applicable to any value of the shock speed analyticaly.

Basic idea of the shock acceleration

1 step of crossing and re-crossing the shock front



Probability that a particle entering to the downstream will eventually return to the upstream $P_R(\mu_0)$

energy gain factor when a particle crosses and re-crosses the shock front $G(\mu_0, \mu)$

Motivation

Two ways to calculate the power-law index

Peacock (1981)

Approximate solution

$$s = 3 - \frac{\ln\langle P_R \rangle}{\langle \ln G \rangle}$$

Vietri (2003)

Exact solution

$$\langle P_R \rangle \langle G^{s-3} \rangle = 1$$

- We examined what is the origin of the difference between Peacock and Vietri.
- When we can use peacock's approximation?

the variance of the energy gain factor's distribution is the origin of this difference

Calculation of return probability

Assumption

- Test particle approximation

  Shock structure is uniform by infinite

  ➡ Return probability is independent from a particle's energy

- Energy conservation

  a particle's energy is conserved when a particle is scattered in the fluid rest frame.

- Diffusive approximation e.g.,Peacock(1981)

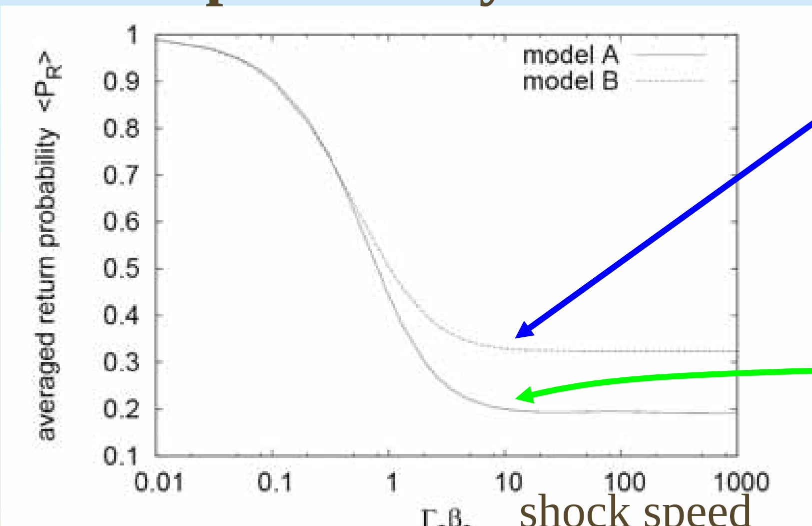
  A particle is scattered many times before it crosses the shock front. We regard behavior as diffusion.

  ➡ Kato & Takahara (01), Vietri (03) include non-diffusive effect

We use Kato & Takahara's formulation to calculate the return probability.

return probability & energy gain factor

return probability



model B

upstream — downstream

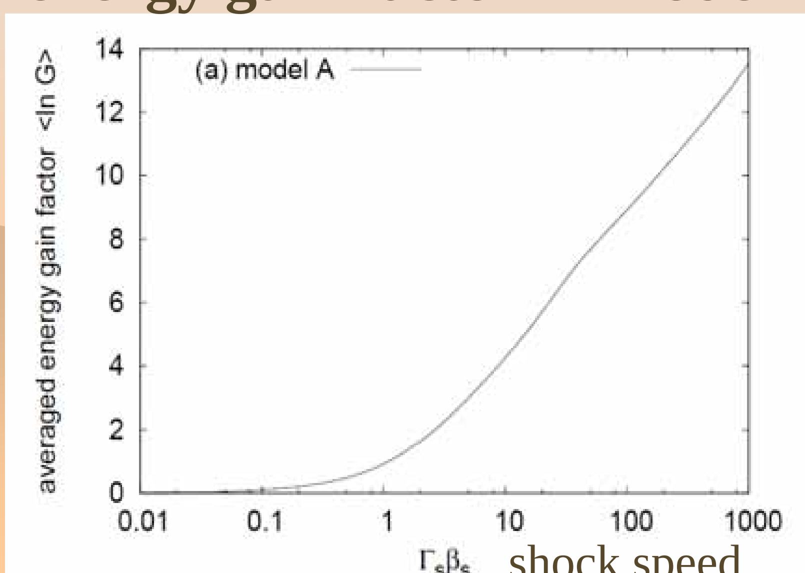
regular deflection — large-angle scattering

model A

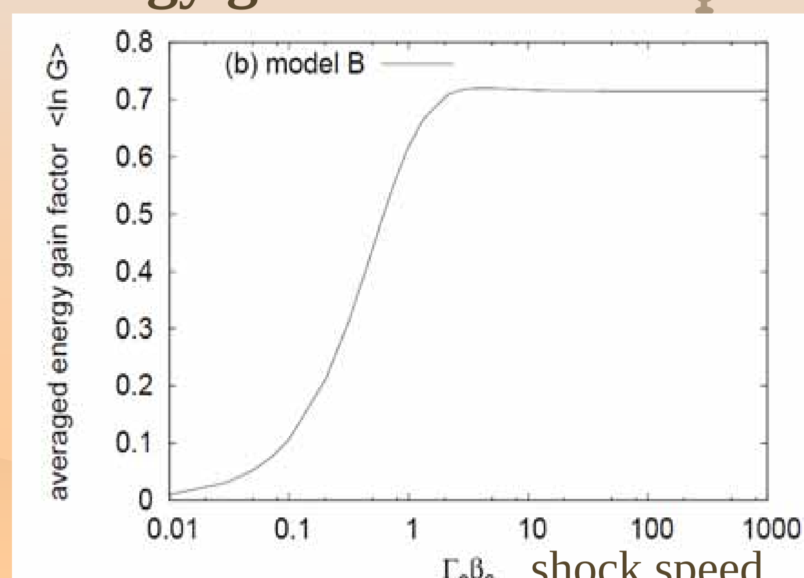
both upstream and downstream

large-angle scattering

energy gain factor in model A



energy gain factor in model B



Calculation of the power-law index (1)

Peacock (1981) : Approximate solution

- un-correlation between distribution of each step
- particles cross and re-cross the shock front many times.

} central limit theorem

number and energy of particles after k steps

$$\frac{N}{N_0} = \langle P_R \rangle^k$$

$$E = E_0 * G_1 * G_2 \cdots * G_k$$

central limit theorem

$$\ln\left(\frac{E}{E_0}\right) = k\langle \ln G \rangle$$

power-law spectrum

$$f(E)d^3\boldsymbol{p} \propto E^{-s} d^3\boldsymbol{p}$$

where

$$s = 3 - \frac{\ln\langle P_R \rangle}{\langle \ln G \rangle}$$

considering the averaged energy gain factor
the effect of the variance is not included

Calculation of the power-law index (2)

Vietri (2003) : Exact solution

- solving a transport equation exactly
- applicable to the case when correlation is strong

This boundary condition gives the power-law index

$$\phi_{ud}(\mu) = \int_{-1}^{-u_d} d\mu_{out} P_u(\mu_{out}, \mu) \times \left(\frac{1 - V_r \mu}{1 - V_r \mu_{out}} \right)^{3-s} \phi_{du}(\mu_{out})$$

rewriting

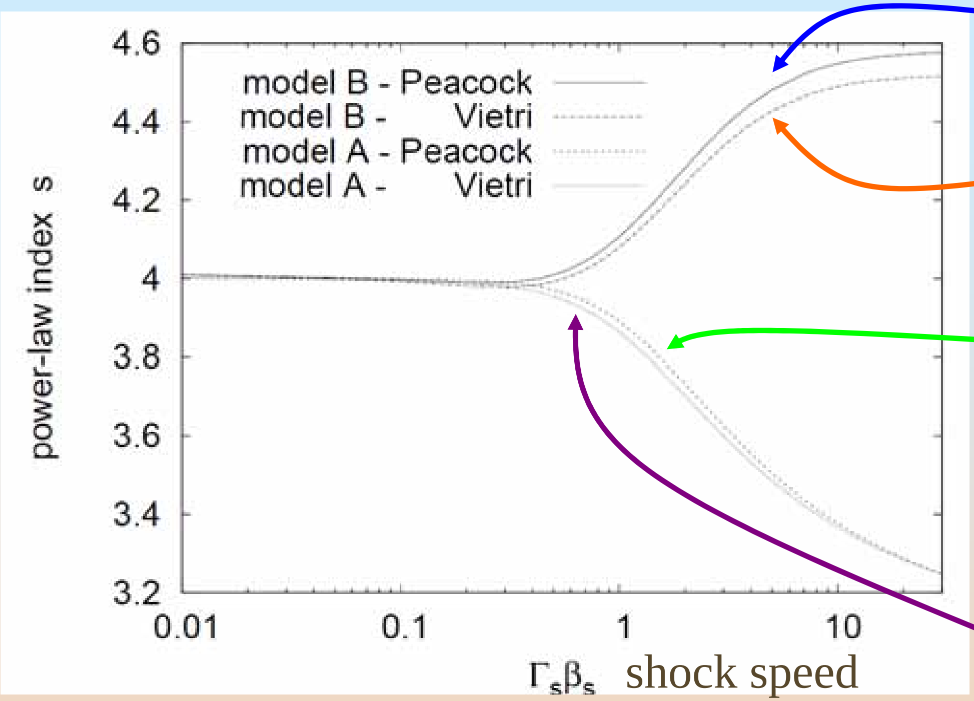
$$\langle P_R \rangle \langle G^{s-3} \rangle = 1$$

In Newtonian limit

$$\langle P_R \rangle \langle G^{s-3} \rangle = 1 \quad \Rightarrow \quad \langle P_R \rangle \langle G \rangle^{s-3} = 1$$

Peacock's approximation

Power-law index



deflection & large-angle scattering Peacock

deflection & large-angle scattering Vietri

large-angle scattering & large-angle scattering Peacock

large-angle scattering & large-angle scattering Vietri

- **large-angle scattering & large-angle scattering**

  return probability & energy gain factor converges

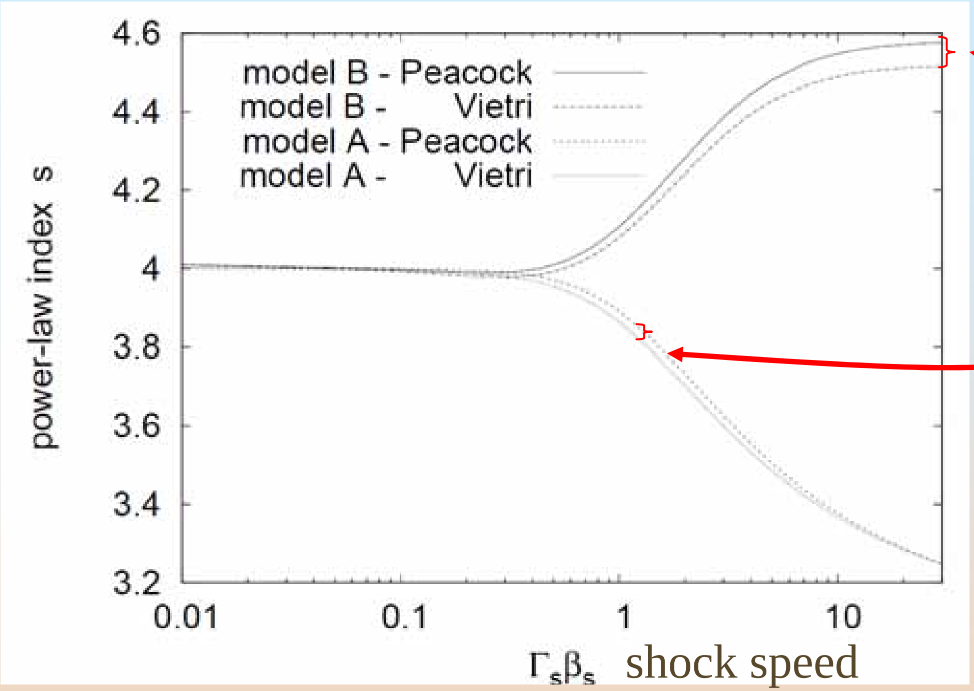
  ⇨ power-law index also **converges** ref., Bednarz & Ostrowski (1998)

- **deflection & large - angle scattering**

  energy gain factor increases monotonically

  ⇨ power-law index **decreases monotonically**

Power-law index



difference between
Vietri's exact solution
&
Peacock's approximate solution

What is the origin of this difference?

Difference of the power-law index

Equation which shows difference between Vietri & Peacock

$$D(\beta_{s1}) = \ln\langle G^{s_1-3}(\beta_{s1})\rangle - (s_1 - 3)\langle \ln G(\beta_{s1})\rangle$$

This equation becomes 0 when variance of the energy gain factor is 0. (e.g. the distribution function of the energy gain factor is delta function)

1) **Two power-law index becomes different as the variance becomes big**

2) **The effect of the variance becomes small as the power-law index becomes small**

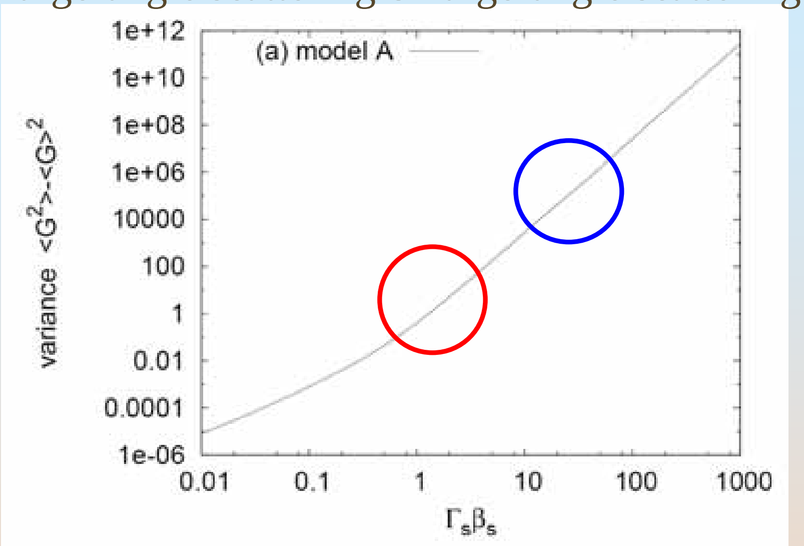
(physical interpretation)

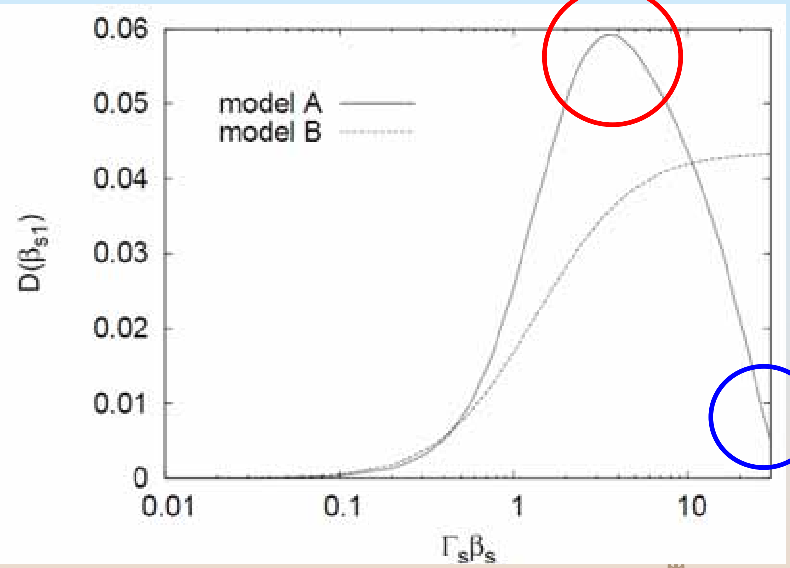
the fraction of particles that are accelerated by the mean energy gain factor increases.

The effect of the variance

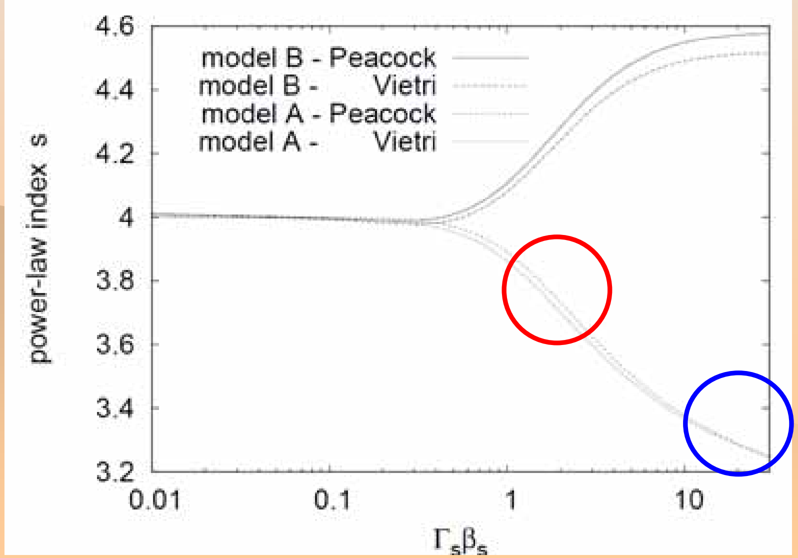
variance model A

large-angle scattering & large-angle scattering





power-law index



model A

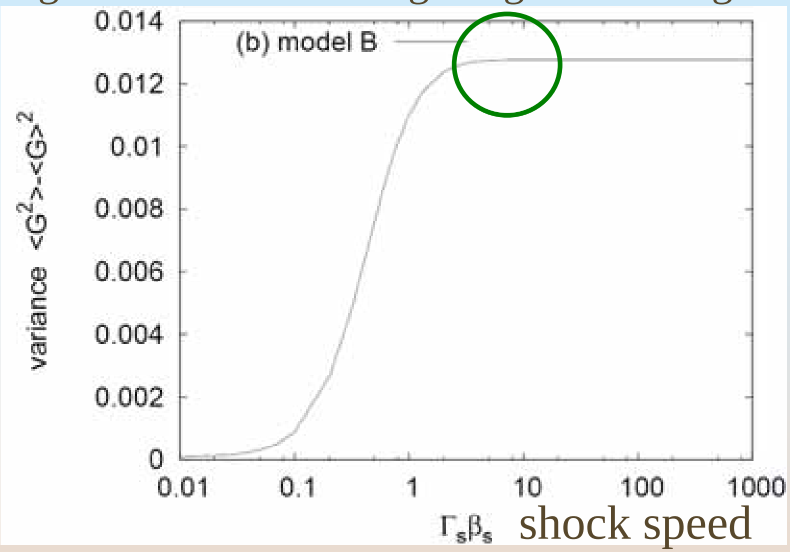
- **strong in the mildly-relativistic shock.**
- **disappearing in the highly-relativistic shock**

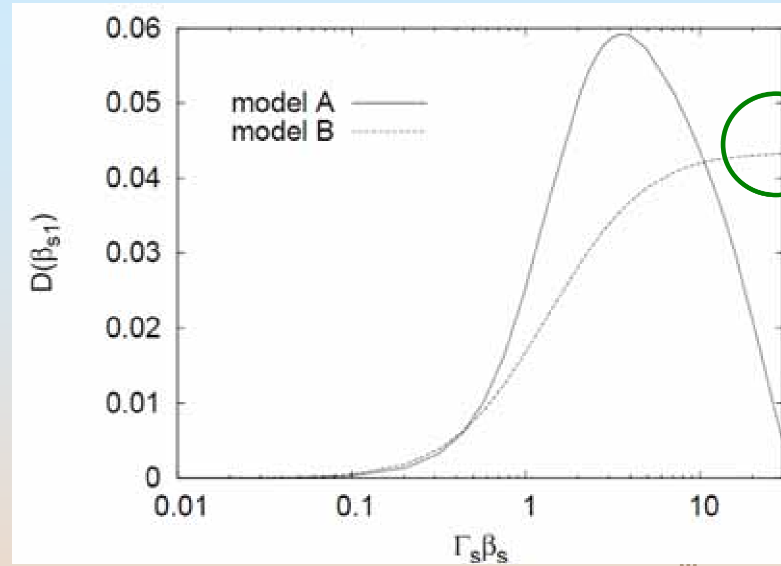
this is because the effect of the variance becomes weak as the power-law index decreases.

The effect of the variance

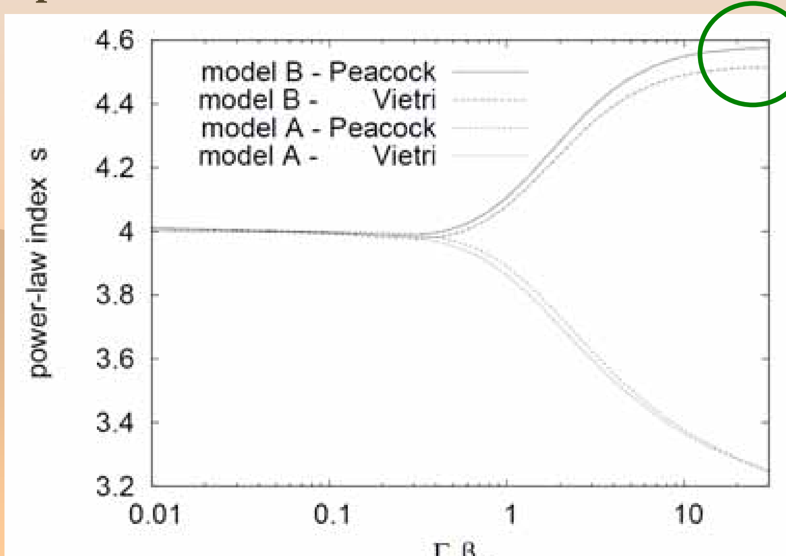
variance model B

regular deflection & large-angle scattering





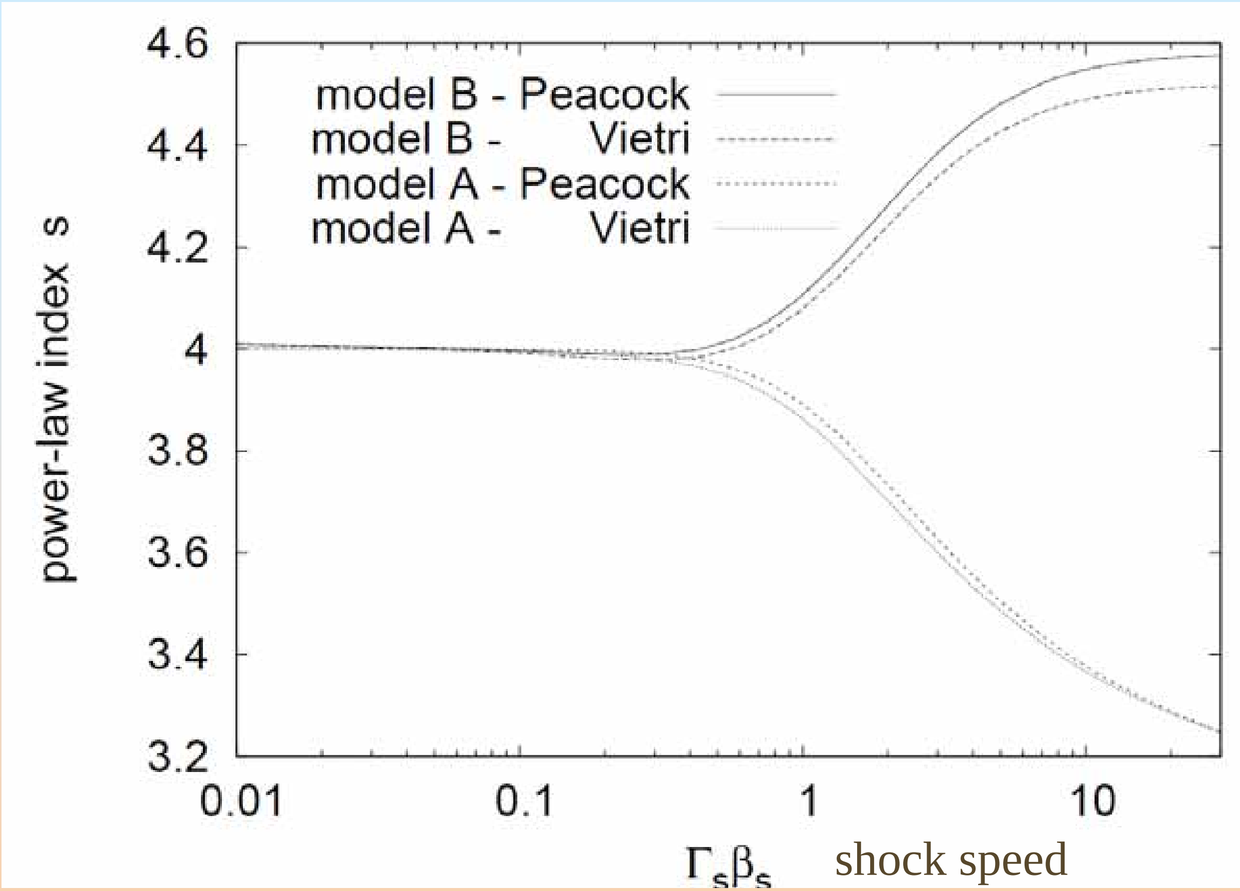
power-law index



model B

•converging in the highly-relativistic shock

Difference of the power-law index



- In model A, the effect of variance disappears because of the the power-law index becomes small
- In model B, difference converges because the variance converges.
- Vietri's index is **harder** than Peacock's one

Conclusion

- Difference between Vietri's exact solution and Peacock's approximation originates from the **variance**
- Peacock's approximation can apply in the case of the non-relativistic shock and the highly-relativistic shock when we consider only the large-angle scattering
- Peacock's approximation is inappropriate when we consider the regular deflection by the effect of the variance.

Fin

appendix A

$$\underbrace{\lambda(u_d+\mu)g(\mu)}_{\text{flux of (k+1) step}} = \int_{-u_d}^{1} d\xi\, Q^T(\xi,\ \mu)\underbrace{(u_d+\xi)g(\xi)}_{\text{flux of k step}}$$

flux of (k+1) step — flux of k step

This kernel shows the correlation between k step and (k+1) step

1) large-angle scattering & large-angle scattering

the kernel is independent of ξ ⇒ we can use Peacock's approximation safely.

Kato & Takahara (2001)

2) regular deflection & large-angle scattering

the kernel depends on ξ ⇒ we can use Peacock' approximation when we consider static condition.

This is because the distribution function doesn't contain information about correlation.

Calculating return probability and flux by Kato & Takahara (2001)

Consider random walk in the shock

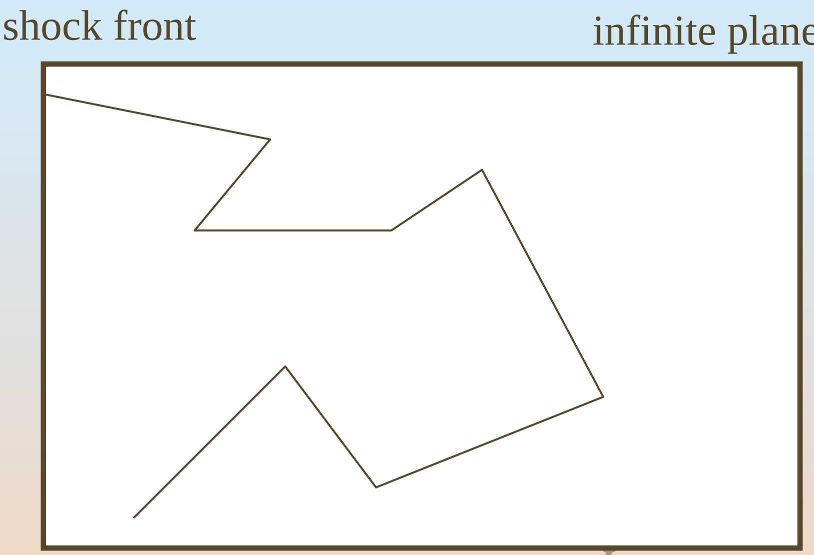
probability density $p(\Delta x) = \frac{1}{\lambda}\exp(-\frac{\Delta x}{\lambda})$

where λ is mean free path ,

Δx is the displacement

•**the large-angle scattering model** both in the up and downstream.

•Un-correlation between distribution of each step.



Consider regular deflection in the upstream

relativistic shock: regular deflection is important

e.g.Gallant & Achterberg (99)

scattering timescale > deflection timescale

We also consider the regular deflection in the upstream.

Strong correlation between distribution of each step